A Unified Theory of Resonance Shifts in Ultrasound Resonance Spectroscopy

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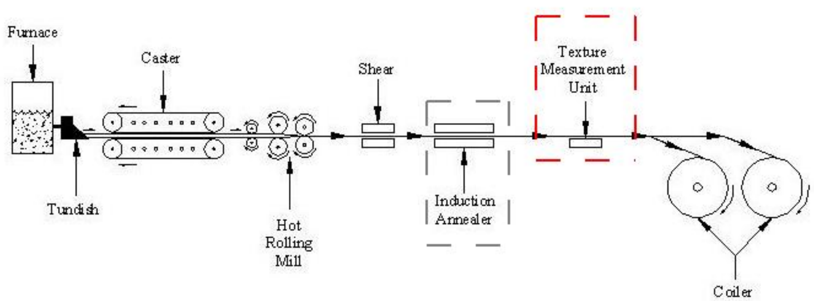
Basic Focus

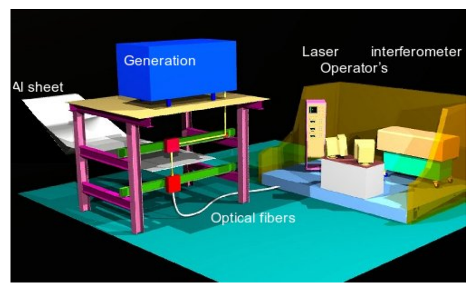
- Rolled metal sheets
- Homogeneous chemical composition
- Aggregates of cubic polycrystals
- Sheets exhibiting orthorhombic symmetry
- Linear elasticity and viscoelasticity

Motivation

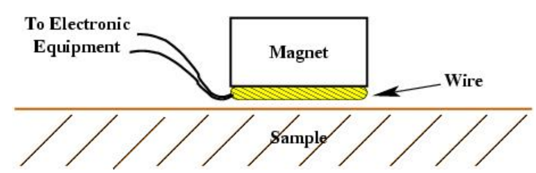


Manufacturing Scenario

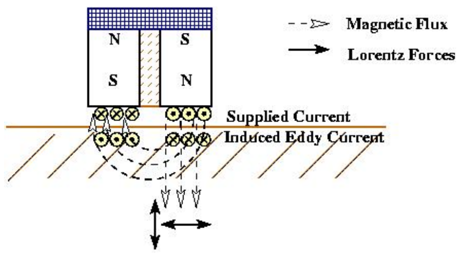
Ultrasound Resonance Spectroscopy (URS)



Resonances can be produced by Electromagnetic Acoustic Transducers (EMATs) or Lasers



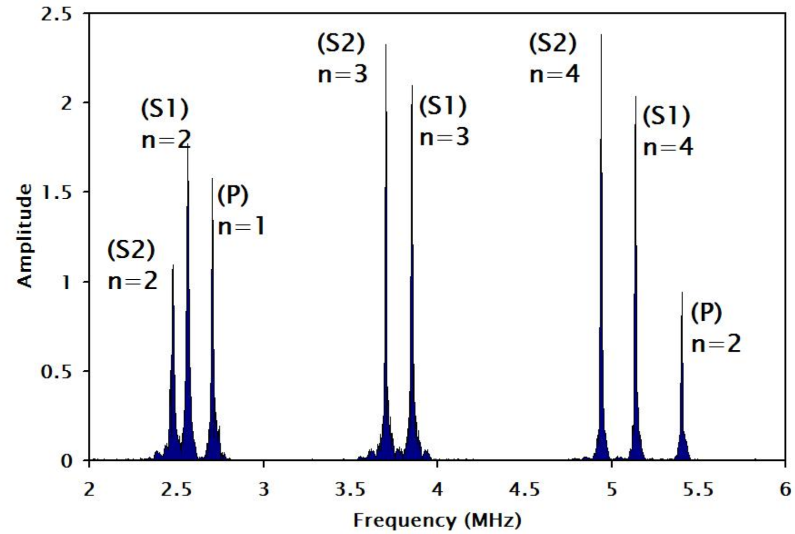
EMATs



No contact required

Wave is generated inside material

1. Burst of current is used ($\backsim 10\mu s$)
2. Interactions between AC current in coil and magnetic field induce a force which displaces material
3. Waves disturb magnetic field inducing current in coil
4. Current is measured by equipment



Isotropic homogeneous elastic material

$$\mathbf{T} = \mathbb{C}[E]$$

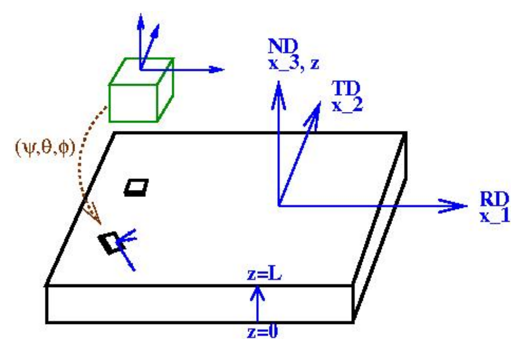
$\mathbf{T}$ = Cauchy Stress
$\mathbb{C}$ = 4th order elasticity tensor, E = infinitesimal strain
Assume $\mathbb{C}$ enjoys major and minor symmetries.

$$\mathbb{C}[E] = \lambda \operatorname{tr}(E)\mathbb{I} + 2\mu E$$

where λ and μ are the Lamé constants

Furthermore, define $u_i(z,t) = \frac{1}{\text{area}(D)} \int_D u_i(x_1, x_2, z, t)\, dA$.

Equations of Motion



$$\mathbf{T} = \mathbb{C}[E]$$

$$\rho \mathbf{u}_{tt} = \operatorname{div} \mathbf{T}$$

with initial/boundary values

$$\frac{\partial u_i}{\partial z}(0,t) = \frac{\partial u_i}{\partial z}(L,t) = 0, \quad u_i(z,0) = g_i(z), \quad \frac{\partial u_i}{\partial t}(z,0) = v_i(z)$$

Solutions

Three equations

$$\frac{\partial^2 u_j}{\partial t^2} = \frac{\mu}{\rho}\nabla^2 u_j \text{ for } j = 1,2 \quad \text{and} \quad \frac{\partial^2 u_3}{\partial t^2} = \frac{\lambda + 2\mu}{\rho}\nabla^2 u_3$$

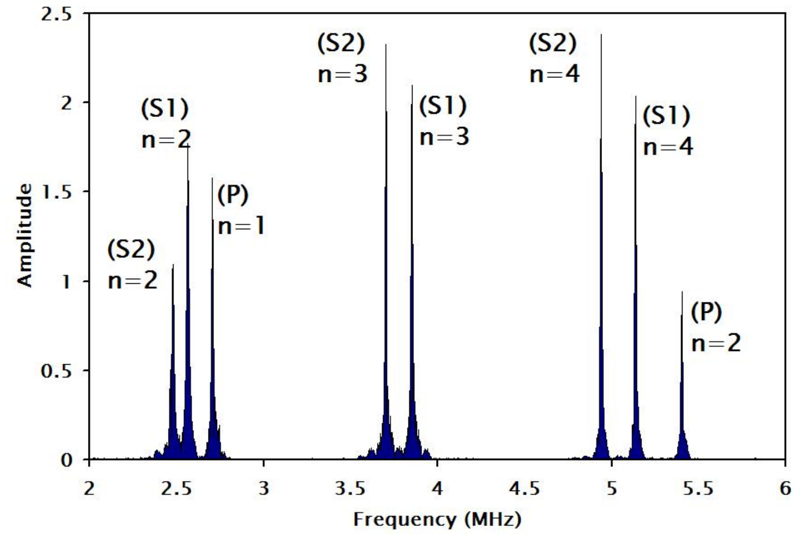
Well-known solutions resulting in resonant frequencies of

$$\omega_n = n\frac{\pi}{L}\sqrt{\frac{C_{\text{iso}}}{\rho}} \text{ where}$$

C_{iso} is μ for case of u_1 and u_2,

and $C_{\text{iso}} = \lambda + 2\mu$ for case u_3

i.e., $f_n = nf_1$



Isotropic material with internal friction

$$\mathbf{T} = \mathbb{C}[E] + \eta[D]$$

$\mathbf{T}$ = Cauchy Stress
$\mathbb{C}$ = 4th order elasticity tensor, E = infinitesimal strain
Assume $\mathbb{C}$ enjoys major and minor symmetries.
η = 4th order effective viscosity tensor, D = stretching tensor

$$\mathbb{C}[E] = \lambda \operatorname{tr}(E)\mathbb{I} + 2\mu E$$

where λ and μ are the Lamé constants.

Furthermore, define $u_i(z,t) = \frac{1}{\text{area}(D)}\int_D u_i(x_1, x_2, z, t)\, dA$.

Equations of Motion

With Internal Friction

$$\rho\frac{\partial u_i}{\partial t^2} = C_{ijk\ell}\frac{\partial^2 u_i}{\partial z^2} + \eta_{ijk\ell}\frac{\partial^3 u_i}{\partial z \partial t^2}$$

with initial/boundary values

$$\frac{\partial u_i}{\partial z}(0,t) = \frac{\partial u_i}{\partial z}(L,t) = 0, \quad u_i(z,0) = g_i(z), \quad \frac{\partial u_i}{\partial t}(z,0) = v_i(z)$$

Solutions

Resonance shift follows from the frequency formula

$$\omega_n = \sqrt{\left(\omega_n^{(0)}\right)^2 - \alpha_n^2}$$

where

$\omega_n^{(0)}$ is frequency in absence of attenuation, i.e., $\omega_n = n\frac{\pi}{L}\sqrt{\frac{C_{\text{iso}}}{\rho}}$

and $\alpha_n := \frac{1}{2\rho}\left(\frac{n\pi}{L}\right)^2 \eta_{ijk\ell}$ is attenuation for the n-th resonance.

Consider both internal friction and texture

$$\mathbf{T} = \mathbb{C}[E] + \eta[D]$$

$\mathbf{T}$ = Cauchy Stress
$\mathbb{C}$ = 4th order elasticity tensor, E = infinitesimal strain
Assume $\mathbb{C}$ enjoys major and minor symmetries.
η = 4th order effective viscosity tensor, D = stretching tensor

$$\begin{aligned} \mathbb{C}(w) &= \mathbb{C}(w_{\text{isotropic}}) + \mathbb{C}'(w - w_{\text{isotropic}}) \\ \mathbb{C}(w)[E] &= \lambda \operatorname{tr}(E)\mathbb{I} + 2\mu E + \beta \mathbf{\Phi}[E] \end{aligned}$$

where λ and μ are the Lamé constants,
β is a material constant, and $\mathbf{\Phi}$ is a 4-th order tensor.

Furthermore, assume $u_i(z,t) = \frac{1}{\text{area}(D)} \int_D u_i(x_1, x_2, z, t)\, dA$.

Details of Constitutive Relationship

In components,

$$\mathbb{C}(w)[E] = \lambda \operatorname{tr}(E)\mathbb{I} + 2\mu E + \beta \mathbf{\Phi}[E]$$

becomes $C_{ijk\ell} = \lambda\delta_{ij}\delta_{k\ell} + \mu\left(\delta_{ik}\delta_{j\ell} + \delta_{i\ell}\delta_{jk}\right) + \alpha\Phi_{ijk\ell}$
where $\Phi_{1122} = W_{400} - \sqrt{70}W_{440}$, $\Phi_{1133} = -4W_{400} + 2\sqrt{10}W_{420}$,
and $\Phi_{2233} = -4W_{400} - 2\sqrt{10}W_{420}$.

The $W_{\ell mn}$ are coefficients in this expansion of the Orientation Distribution Function w:

$$w(\psi,\theta,\phi) = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\sum_{n=-\ell}^{\ell} W_{\ell mn} Z_{\ell mn}(\cos\theta) e^{-im\psi} e^{-in\phi}$$

where we follow Roe's convention [1960's] here.

- Let the $W_{\ell mn}$ coefficients vary through the thickness z

Texture and Internal Friction

Texture coefficients vary through the thickness

$$\mathbb{C}(w(z)) = \mathbb{C}(w_{\text{isotropic}}) + \mathbb{C}'(w(z) - w_{\text{isotropic}})$$

- Equation of Motion

$$\rho\frac{\partial^2 u_i}{\partial t^2} = \frac{\partial}{\partial z}\left(C_{ijk\ell}(z)\frac{\partial u_i}{\partial z} + \eta_{ijk\ell}\frac{\partial^2 u_i}{\partial z \partial t}\right)$$

with initial/boundary values

$$\frac{\partial u_i}{\partial z}(0,t) = \frac{\partial u_i}{\partial z}(L,t) = 0, \quad u_i(z,0) = g_i(z), \quad \frac{\partial u_i}{\partial t}(z,0) = v_i(z)$$

Equations of Motion

Exercising weakly textured assumption:

$$\mathbb{C}(w(z)) = \mathbb{C}_{\text{iso}} + \varepsilon \mathbb{C}_{\text{tex}}(z)$$

Sheet with internal friction and inhomogeneous texture

For each wave mode $i = 1,2,3,$

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial z}\left(C_{\text{iso}} \frac{\partial u}{\partial z} + \eta \frac{\partial^2 u}{\partial z \partial t} \right) + \varepsilon \frac{\partial}{\partial z}\left(C_{\text{tex}}(z) \frac{\partial u}{\partial z} \right)$$

with initial/boundary values

$$\frac{\partial u}{\partial z}(0,t) = \frac{\partial u}{\partial z}(L,t) = 0, \quad u(z,0) = g(z), \quad \frac{\partial u}{\partial t}(z,0) = v(z)$$

A Perturbation Scheme

For each wave mode $i = 1,2,3,$

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial z}\left(C_{\text{iso}} \frac{\partial u}{\partial z} + \eta \frac{\partial^2 u}{\partial z \partial t} \right) + \varepsilon \frac{\partial}{\partial z}\left(C_{\text{tex}}(z) \frac{\partial u}{\partial z} \right)$$

can be thought of as

$$\rho \frac{\partial^2 u}{\partial t^2} = \mathscr{A} u + \varepsilon \mathscr{C} u.$$

Perturbation Scheme (continued)

Writing $u = u^{(0)} + \varepsilon u^{(1)} + \varepsilon^2 u^{(2)} + \ldots$, then

1. $u^{(0)}$ satisfies $\rho \frac{\partial^2 u^{(0)}}{\partial t^2} = \mathscr{A} u^{(0)}$ and
2. $u^{(1)}$ satisfies $\rho \frac{\partial^2 u^{(1)}}{\partial t^2} = \mathscr{A} u^{(1)} + \varepsilon \mathscr{C} u^{(0)}$,
3. $u^{(2)}$ satisfies $\rho \frac{\partial^2 u^{(2)}}{\partial t^2} = \mathscr{A} u^{(2)} + \varepsilon \mathscr{C} u^{(1)}$,
4. $\ldots$, etc.

with appropriate boundary conditions:

1. $\frac{\partial u^{(0)}}{\partial z}(0,t) = \frac{\partial u^{(0)}}{\partial z}(L,t) = 0, u^{(0)}(z,0) = f(z), \frac{\partial u^{(0)}}{\partial t}(z,0) = g(z),$
2. $\frac{\partial u^{(1)}}{\partial z}(0,t) = \frac{\partial u^{(1)}}{\partial z}(L,t) = 0, \quad u^{(1)}(z,0) = 0, \quad \frac{\partial u^{(1)}}{\partial t}(z,0) = 0,$
3. $\frac{\partial u^{(2)}}{\partial z}(0,t) = \frac{\partial u^{(2)}}{\partial z}(L,t) = 0, \quad u^{(2)}(z,0) = 0, \quad \frac{\partial u^{(2)}}{\partial t}(z,0) = 0,$
4. $\ldots$, etc.

Resonance Shifts

Inhomogeneous Texture and Homogeneous Viscosity

$$\omega_n = \sqrt{\left(\omega_n^{(0)}\right)^2 - \alpha_n^2} + \varepsilon \frac{-1}{\rho L \sqrt{\left(\omega_n^{(0)}\right)^2 - \alpha_n^2}} \int_0^L \cos\frac{n\pi z}{L} \frac{d}{dz}\left(C_{\text{tex}}(z) \frac{d}{dz} \cos\frac{n\pi z}{L} \right) dz$$

$\omega_n^{(0)}$ is frequency in absence of attenuation
and α_n is the attenuation

Resonance Shifts

Inhomogeneous Texture and Homogeneous Viscosity

If α_n is small
$$\omega_n = \omega_n^{(0)} - \frac{\alpha_n^2}{2\omega_n^{(0)}} + \varepsilon \frac{-1}{\rho L \omega_n^{(0)}} \int_0^L \cos\frac{n\pi z}{L} \frac{d}{dz}\left(C_{\text{tex}}(z) \frac{d}{dz} \cos\frac{n\pi z}{L}\right) dz$$

Model C_{tex} as symmetric about $z = L/2$, then the frequency shift can be modeled as follows:

$$\frac{f_n}{n} - f_0 = -L\sqrt{\frac{\rho}{C_{\text{iso}}}}\left(\frac{\alpha_n}{2n\pi}\right)^2 - \varepsilon\frac{1}{2L^2\rho}\sqrt{\frac{\rho}{C_{\text{iso}}}}\int_0^{L/2} C_{\text{tex}}(z)\cos\frac{2n\pi z}{L}\, dz$$

where $\omega_n = 2\pi f_n$ and $f_0 := \frac{1}{2L}\sqrt{\frac{C_{\text{iso}}}{\rho}}$.

Resonance Shifts (con't)

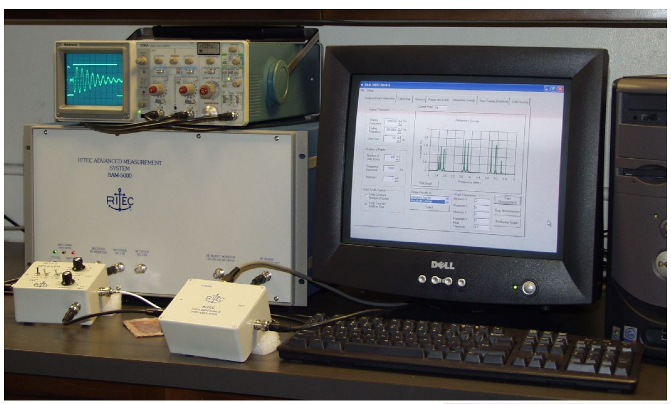
A Resonance Shift Formula

$$\left(\frac{f_n^{(S1)}}{n} - f_0^{(S1)}\right) + \left(\frac{f_n^{(S2)}}{n} - f_0^{(S2)}\right) =$$
$$\frac{-1}{8\overline{f_0}(n\pi)^2}\left((\alpha_n^{(S1)})^2 + (\alpha_n^{(S2)})^2\right) + \frac{2\beta\overline{f_0}}{n^2\pi^2\mu}\left(b + \frac{a}{2} - \frac{3a}{n^2\pi^2}\right)$$

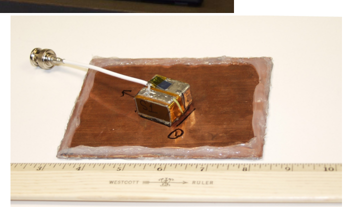
Experiment to check the resonance shift formulas.

- Using model $W_{400}(z) = a\left(\frac{z}{L} - \frac{1}{2}\right)^4 + b\left(\frac{z}{L} - \frac{1}{2}\right)^2 + c$.
- This formula requires surface texture to be known.
- Measure $f_n^{(S1)}$, $f_n^{(S2)}$, and high resonances to determine f_0.
- Use $\beta/\mu = -3.929$ per Huang and Man [2003].

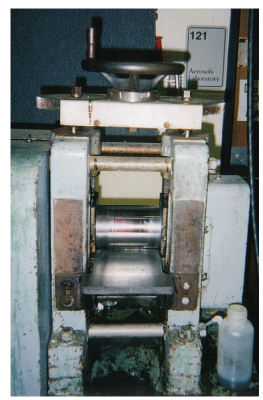
Experiments



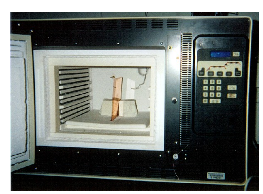
- Measurements were made on C11000 copper (ETP) with Ritec RAM-5000 system
- In-house EMATs constructed for experiments

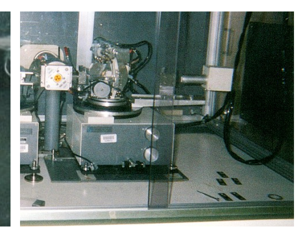


Sample Preparation



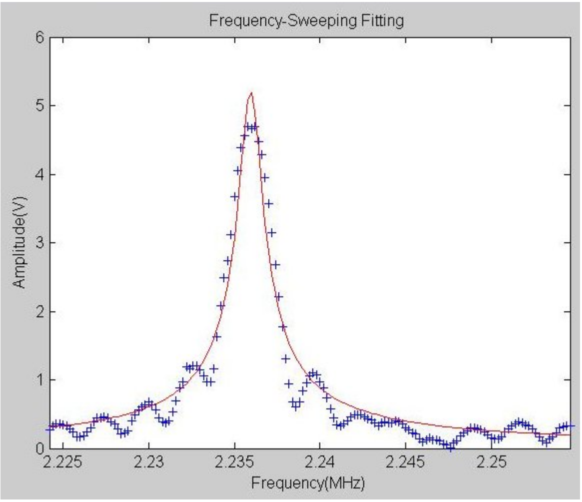
1. As received sample cold-rolled to ≈ 0.86 mm (95% thickness reduction)
2. Annealed for 30 minutes at 600° F.
3. XRD measurements were taken at surface, $\frac{1}{4}$ depth, and $\frac{1}{2}$ depth as reference values.





Attenuation Recovery

A Lorentz Line Shape function $|A|^2 = \dfrac{c}{\alpha^2 + 4\pi^2 (f - \tilde{f})^2}$ is fitted to each measured resonance.



A =Amplitude
c =constant,
α =attenuation,
f =frequency, and
$\tilde{f}$ =resonance frequency

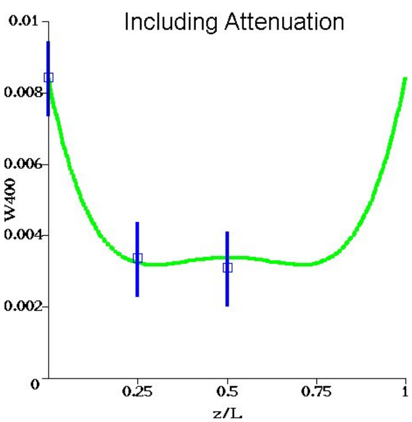
Using the resonance shift formula

$$\left(\frac{f_n^{(S1)}}{n} - f_0^{(S1)}\right) + \left(\frac{f_n^{(S2)}}{n} - f_0^{(S2)}\right) = \frac{-1}{8\overline{f_0}(n\pi)^2}\left((\alpha_n^{(S1)})^2 + (\alpha_n^{(S2)})^2\right) + \frac{2\beta \overline{f_0}}{n^2\pi^2\mu}\left(b + \frac{a}{2} - \frac{3a}{n^2\pi^2}\right)$$

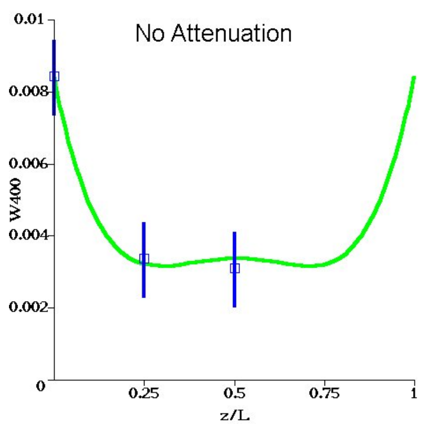
- Using model $W_{400}(z) = a\left(\frac{z}{L} - \frac{1}{2}\right)^4 + b\left(\frac{z}{L} - \frac{1}{2}\right)^2 + c$.
- This formula requires surface texture to be known.

Results

Attenuation effects not significant

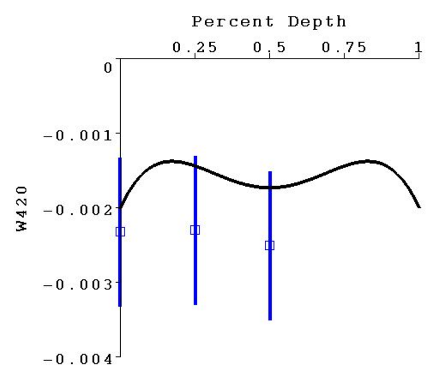


$W_{400}(z) = 0.12039\left(\frac{z}{L} - \frac{1}{2}\right)^4 - 0.01012\left(\frac{z}{L} - \frac{1}{2}\right)^2 + 0.00344$



$W_{400}(z) = 0.12070\left(\frac{z}{L} - \frac{1}{2}\right)^4 - 0.01013\left(\frac{z}{L} - \frac{1}{2}\right)^2 + 0.00342$

Another C11000 copper sample:



- More experiments needed

Small texture gradient

Conclusions and Further Work

- Experimental data is consistent with formula for resonance shifts for these particular samples.
- When texture gradient is strong and attenuation small, theory may be useful to detect through-thickness texture gradients.

Present theory: $\alpha_n \propto f_n^2 \longrightarrow$ Future experimental considerations:

- Frequency range of these experiments too low for attenuation to be a factor.
- Need measurements at higher frequencies to verify the resonance shift formulas in presence of large attenuation.

Further theoretical considerations:

- Portion of theory on ultrasonic attenuation needs improvement to allow for effects of grain scattering.
- Perhaps a non-linear theory for attenuation is more appropriate.

Acknowledgements

NRC

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SNP Organizers

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