

Quantitative relationships between  
stress distributions, microstructure, and  
high strain rate performance of advanced ceramics:  
an interim report

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WMRD



# Project

## Contribute to fundamental theory

**Conditions:** high strain rate, high stress, large strain

**Events:** cracking and failure

**Materials:** armor ceramics

## Develop quantitative relationships

between response of ceramics and

- **microstructures** (texture, grain size, grain shape)
- **stress distributions**



# Today's Goals

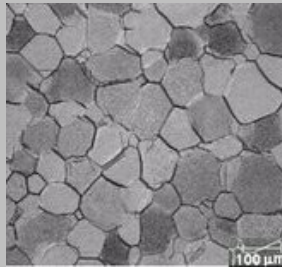
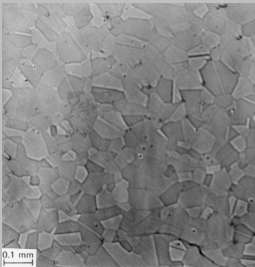
Report on progress made in understanding affect of texture on internal stress and clarifying internal stress distributions

## Outline

- 1 Crystallographic Texture
  - Relationship to Stress
  - Thermo-anisotropic Elasticity
  - Visualization and  $\sigma_{ij}^2$
  - In Progress
- 2 Stress Distributions
  - Papers Reviewed Last Year
  - Research Direction
  - In Progress



# Texture



Two different AION samples, scale 0.1 mm

- $\sigma = \mathbb{C}(w)[E]$
- Elastic constants  $\mathbb{C}_{ijkl}$
- Orientation distribution function  $w$



# Thermo-anisotropic Elasticity

## Thermal contractions lead to fracture in brittle materials

- Equations of thermo-anisotropic elasticity

$$\nabla \cdot \sigma = 0 \quad \text{equilibrium}$$

$$\nabla \cdot h = 0 \quad \text{balance of energy}$$

$$\sigma = \mathbb{C}[E] - \beta T \quad \text{stress-strain law}$$

$$h = -\kappa \nabla T \quad \text{heat conduction}$$

$\beta$  = thermal expansion tensor,  $\kappa$  = thermal conductivity tensor,  
 $h$  = heat flux,  $T$  = temperature

- Solved in 2-D by T.C.T. Ting (1996) using Stroh's formalism



# Visualization

What does the internal stress look like?

## OOF<sup>2</sup>

- **O**bject **O**riented **F**inite element method in **2**-dimensions
- Specifically designed for use on actual micrographs
- Heat & force balance equations ( $\nabla \cdot \text{flux} = \text{applied force}$ )  
Plane flux equations (out of plane components of flux = 0)
- Can be extended using **C++** or **Python**



## OOF2

The screenshot displays the OOF2 software interface, which is divided into several windows:

- OOF2 (Main Window):** Contains a menu bar (File, Settings, Windows, Help) and a toolbar. The left sidebar shows a tree view of material properties under "Mechanical", "Thermal", and "Electric". The main area shows the "Material" properties for "yellowMaterial", including "Orientation: yellowOrientation", "Mechanical: Elasticity: Anisotropic: Cubic: grahamConstants", "Color: yellowColor", and "Thermal: Conductivity: Anisotropic: Cubic: surmetConstant".
- OOF2 Graphics 1 (Graphics Window):** Shows a stress distribution plot on a grid. The plot is color-coded from black (low stress) to yellow (high stress). The "Zoom" factor is set to 1.5. A vertical color scale on the right indicates a value of 66.9.
- OOF2 Messages 1 (Message Window):** Displays the following text:

```
File Windows
Save...
[Error] [Query] [Report] [Warning] [Log]
OOF2: OOF2.PLOT.TECHNICAL
{field=Temperature, field_component='', equation=Heat_Eqn, eqn_component='', profile=ConstantProfile
{value=5.0}, boundary='top'}}
OOF: Solver.Solve(mesh='sarendipity.png;skeleton=mesh', solver=LinearDriver{method=CGSolver
(max_iterations=1000, tolerance=1e-13, preconditioner=ILUPreconditioner())})
Made 420x420 stiffness matrix
CG converged!
residual = 5.15786e-14 number of completed iterations = 55
OOF: Graphics_1.Settings.Zoom.Fill_Window()
```

available at <http://www.ctcms.nist.gov/oof>



# In Progress

## Visualize internal stress in AION

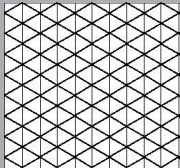
- Implement thermo-anisotropic elasticity equations
- Use material constants, orientation data, and micrograph from actual sample
- Use OOF2
- Extend as necessary



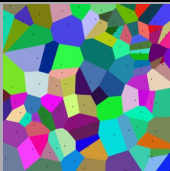


# Stress Distributions

## Papers Reviewed Last Year



① X.H. Zhang, *et al.* (2004)



② D. Zhang, *et al.* (2005)

Split Hopkinson Pressure Bar  
experiments and simulation

③ Tasdemirci and Hall (2005)



## Last Year (con't)

Maximum stress alone is not best predictor of damage.  
Stress inhomogeneity is important too.

- After microcracking but before large cracks: high stresses shift rapidly from one location to another.
- As compressive stress increased: heterogeneity of stress states increased.



# Research Direction

## Idea

- High stress next to low stress is a *critical event*
- Heterogeneity in internal stress states can predict imminent crack formation

⇒ Use internal stress state to predict response of ceramics to high strain rate, high stress, large strain conditions

## Hypothesis

The heterogeneity of internal stress distributions, not simply exceeding a maximum stress, leads to cracking.



# In Progress

Characterize stress states using fractal dimension

## Definition (fractal dimension: box-counting dimension)

Given a self-similar object of  $N$  parts scaled by the ratio  $r$  from the whole, its fractal dimension is

$$D = \frac{\log N}{\log (1/r)}.$$

## Multifractal formalism

- When different subsets of the object exhibit different fractal dimensions the object is considered to be **multifractal**
- The **singularity spectrum** fully describes a multifractal object



# Summary

## Effects of Texture

- 1 Preferred crystallographic orientation and mechanical anisotropy of individual crystals affects stress response
- 2 Working on numerical solution to thermo-anisotropic equations using OOF2 and “real” data to visualize affects of anisotropy on internal stresses under compression

## Stress Distributions

- 3 Simulations reported by others show stress heterogeneities prior to cracking
- 4 Multifractal formalism under investigation as method to characterize heterogeneous stress states prior to cracking.



# Acknowledgements

## NRC

This research is made possible by a ARL-USMA Davies Fellowship, a postdoctoral fellowship through the NRC.

## ARL Advisors

Thanks to Thomas Wright and James McCauley for their advice and support.

## Many Others

Jeff Swab, Amy H Erickson, and other colleagues



# References



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D. Zhang, M.S. Wu, and R. Feng

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X.H. Zhang, F. Rong, Z.K. Jia, *et al.*

Coupling effects of heterogeneity and stress fluctuation on rupture.

*Theor. & Appl. Fracture Mechanics*, 41:381-389, 2004.



# -End of Slides-

## Quantitative relationships between stress distributions, microstructure, and high strain rate performance of advanced ceramics: an interim report

### Project Goal

To better predict fracture in advanced ceramics under high strain rate, high compressive stress, and large strains by studying microstructure and internal stress distributions

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