Quantitative relationships between stress distributions, microstructure, and high strain rate performance of advanced ceramics: an interim report

Leigh L. Noble, Davies Fellow
leigh.noble@usma.edu

ARL Sponsors:
James W. McCauley & Thomas W. Wright
WMRD

Project

Contribute to fundamental theory

Conditions: high strain rate, high stress, large strain
Events: cracking and failure
Materials: armor ceramics

Develop quantitative relationships

between response of ceramics and

- **microstructures** (texture, grain size, grain shape)
- **stress distributions**

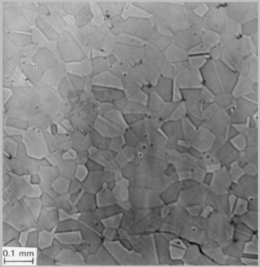
Today's Goals

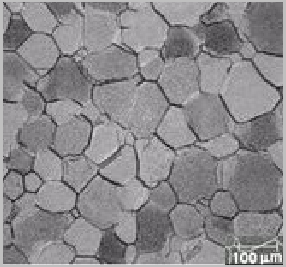
Report on progress made in understanding affect of texture on internal stress and clarifying internal stress distributions

Outline

1. Crystallographic Texture
   - Relationship to Stress
   - Thermo-anisotropic Elasticity
   - Visualization and **`OOF2`**
   - In Progress
2. Stress Distributions
   - Papers Reviewed Last Year
   - Research Direction
   - In Progress

Texture





Two different AlON samples, scale 0.1 mm

- $\sigma = \mathbb{C}(w)[E]$
- Elastic constants $\mathbb{C}_{ijk\ell}$
- Orientation distribution function w

Thermo-anisotropic Elasticity

Thermal contractions lead to fracture in brittle materials

- Equations of thermo-anisotropic elasticity

$$\begin{aligned} \nabla \cdot \sigma &= 0 && \text{equilibrium} \\ \nabla \cdot h &= 0 && \text{balance of energy} \\ \sigma &= \mathbb{C}[E] - \mathcal{B}T && \text{stress-strain law} \\ h &= -\boldsymbol{\kappa}\nabla T && \text{heat conduction} \end{aligned}$$

$\mathcal{B} =$ thermal expansion tensor, $\boldsymbol{\kappa} =$ thermal conductivity tensor, $h =$ heat flux, $T =$ temperature

- Solved in 2-D by T.C.T. Ting (1996) using Stroh's formalism

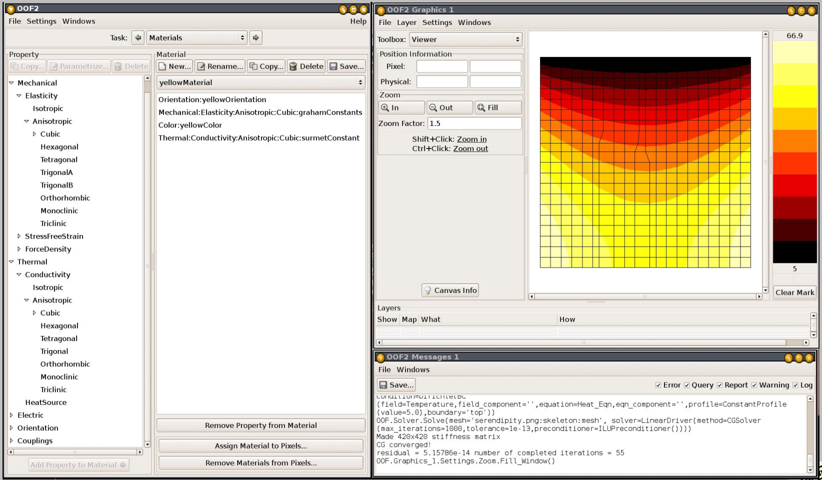
Visualization

What does the internal stress look like?

OOF2

- **O**bject **O**riented **F**inite element method in **2**-dimensions
- Specifically designed for use on actual micrographs
- Heat & force balance equations ($\nabla \cdot \text{flux} = \text{applied force}$)
  Plane flux equations ($\text{out of plane components of flux} = 0$)
- Can be extended using **`C++`** or **`Python`**

OOF2



available at **`http://www.ctcms.nist.gov/oof`**

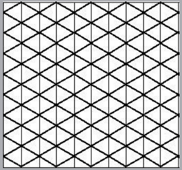
In Progress

Visualize internal stress in AlON

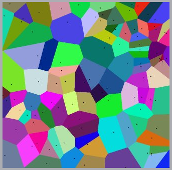
- Implement thermo-anisotropic elasticity equations
- Use material constants, orientation data, and micrograph from actual sample
- Use **`OOF2`**
- Extend as necessary

Stress Distributions

Papers Reviewed Last Year



1. X.H. Zhang, *et al.* (2004)



2. D. Zhang, *et al.* (2005)

Split Hopkinson Pressure Bar experiments and simulation

3. Tasdemirci and Hall (2005)

Last Year (con't)

Maximum stress alone is not best predictor of damage. Stress inhomogeneity is important too.

- After microcracking but before large cracks: high stresses shift rapidly from one location to another.
- As compressive stress increased: heterogenity of stress states increased.

Research Direction

Idea

- High stress next to low stress is a *critical event*
- Heterogeneity in internal stress states can predict imminent crack formation

$\Longrightarrow$ Use internal stress state to predict response of ceramics to high strain rate, high stress, large strain conditions

Hypothesis

The heterogeneity of internal stress distributions, not simply exceeding a maximum stress, leads to cracking.

In Progress

Characterize stress states using fractal dimension

Definition (fractal dimension: box-counting dimension)

Given a self-similar object of N parts scaled by the ratio r from the whole, its fractal dimension is

$$D = \frac{\log N}{\log(1/r)}.$$

Multifractal formalism

- When different subsets of the object exhibit different fractal dimensions the object is considered to be **multifractal**
- The **singularity spectrum** fully describes a multifractal object

Summary

Effects of Texture

1. Preferred crystallographic orientation and mechanical anisotropy of individual crystals affects stress response
2. Working on numerical solution to thermo-anisotropic equations using **`OOF2`** and “real” data to visualize affects of anisotropy on internal stresses under compression

Stress Distributions

3. Simulations reported by others show stress heterogeneities prior to cracking
4. Multifractal formalism under investigation as method to characterize heterogeneous stress states prior to cracking.

Acknowledgements

NRC

This research is made possible by a ARL-USMA Davies Fellowship, a postdoctoral fellowship through the NRC.

ARL Advisors

Thanks to Thomas Wright and James McCauley for their advice and support.

Many Others

Jeff Swab, Amy H Erickson, and other colleagues

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Project Goal

To better predict fracture in advanced ceramics under high strain rate, high compressive stress, and large strains by studying microstructure and internal stress distributions

leigh.noble@usma.edu
MADN-MATH, USMA
West Point, NY 10996-1905 USA