Apparently I mis-numbered the problems!
There is NO #25 or 26

27) The MVT says if f is a differentiable function on an interval $[a,b]$, then there exists a number c $(a<c<b)$ such that

$$f'(c)=\frac{f(b)-f(a)}{b-a}.$$

Ms. Fastdriver should get a citation if her instantaneous velocity is over the speedlimit.

Her average velocity is $\frac{(113.8-1.2)\text{mi}}{(2{:}50\text{pm}-1{:}30\text{pm})}=\frac{112.6\text{mi}}{4/3\text{ hr}}$

$=84.45\text{mph}$

By the MVT, Ms. Fastdriver's instantaneous velocity must have been equal to 84.45mph at some time during that trip. This is clearly over the speed limit, so NJ Transit should issue a ticket.

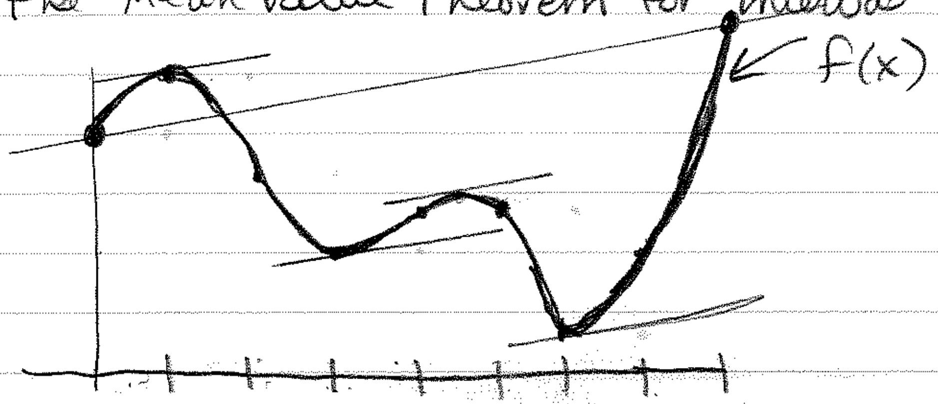
28) By the MVT, there is some time t_* so that

$v'(t_*)=\frac{v(b)-v(a)}{b-a}$ where $a=2{:}00\text{pm}$ + $b=2{:}10\text{pm}$,

i.e. $v'(t_*)=\frac{50\text{mph}-30\text{mph}}{10\text{min}\cdot\frac{1\text{hr}}{60\text{min}}}=\frac{20\text{ mi/h}}{1/6\text{ h}}=120\text{mi/hr}^2.$

So by the MVT there is some time t_* between 2:00pm and 2:10pm so that the acceleration is 120 mi/hr² at the time t_*.

29) Use the graph of f to estimate the values of c that satisfy the conclusion of the Mean Value Theorem for interval $[0, 8]$.



My curve is a little sloppy, but the "c"-values that satisfy the MVT are ≈ $c = 1, 3, 4.5, 6$.
Those are the places where $f'(c) \approx \dfrac{f(8)-f(0)}{8-0}$.

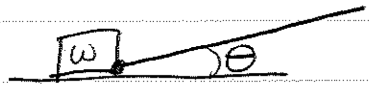
30) Let $g(t)$ be the position of runner 1 at time t.
Let $h(t)$ " " " " 2 " " ".
Define $f(t) = g(t) - h(t)$ be the distance between the runners at time t.
If they finish the race in a tie (+start @ same time), this means $f(t_f) = g(t_f) - h(t_f) = 0$ and $f(t_0) = g(t_0) - h(t_0) = 0$ (where t_f is the finish time and t_0 is the start time).

Since $f(t)$ is a differentiable function, we know there must be some t_* so that between t_0 + t_f

$$f'(t_*) = \frac{f(t_f) - f(t_0)}{t_f - t_0}.$$

But since $f(t_f) - f(t_0) = 0 - 0 = 0$, this means $f'(t_*) = 0$. Therefore $g'(t_*) - h'(t_*) = 0$.
This means $g'(t_*) = h'(t_*)$, i.e. their velocities must be the same at some time.

36)



W = weight of object, CONST
θ = angle between rope + plane, VAR
F = Force, VAR
μ = coefficient of friction, CONST
Given: μ is positive.

Show F is minimized when $\tan\theta = \mu$ subject to $0 \le \theta \le \pi/2$.

$$F = \frac{\mu W}{\mu\sin\theta + \cos\theta}$$

$\boxed{\text{This is an optimization problem, not related rates. Take derivative WRT } \theta\text{, not } t.}$

$$F(\theta) = \frac{\mu W}{\mu\sin\theta + \cos\theta}$$

$$F'(\theta) = \frac{(\mu\sin\theta + \cos\theta)\overbrace{\frac{d}{d\theta}(\mu W)}^{0} - \mu W\frac{d}{d\theta}(\mu\sin\theta + \cos\theta)}{(\mu\sin\theta + \cos\theta)^2}$$

$$= \frac{-\mu W(\mu\cos\theta - \sin\theta)}{(\mu\sin\theta + \cos\theta)^2}$$

Find critical numbers

$F'(\theta) = 0$, i.e. $\frac{-\mu W(\mu\cos\theta - \sin\theta)}{(\mu\sin\theta + \cos\theta)^2} = 0$

$\Rightarrow -\mu W(\mu\cos\theta - \sin\theta) = 0$

$-\mu W = 0$ (NEVER) or $\mu\cos\theta - \sin\theta = 0$

$\mu\cos\theta = \sin\theta$

$\mu = \frac{\sin\theta}{\cos\theta}$

$\mu = \tan\theta$, i.e. $\theta = \tan^{-1}\mu$

This is the only critical number between $0 + \pi/2$, so we need to show it is the minimum

$\longrightarrow$

36) (cont)

Let's try the 2nd derivative test:

$$F''(\theta)=\frac{(\mu\sin\theta+\cos\theta)^2\{-\mu w(-\mu\sin\theta-\cos\theta)\} - (-\mu w(\mu\cos\theta-\sin\theta))\{2(\mu\sin\theta+\cos\theta)(\mu\cos\theta-\sin\theta)\}}{(\mu\sin\theta+\cos\theta)^4}$$

$$=\frac{(\mu\sin\theta+\cos\theta)^2\{\mu w(\mu\sin\theta+\cos\theta)\} + 2\mu w(\mu\cos\theta-\sin\theta)^2(\mu\sin\theta+\cos\theta)}{(\mu\sin\theta+\cos\theta)^4}$$

$$=\frac{(\mu\sin\theta+\cos\theta)\{\mu w(\mu\sin\theta+\cos\theta)^2+2\mu w(\mu\cos\theta-\sin\theta)^2\}}{(\mu\sin\theta+\cos\theta)^4}$$

Now we have three pieces, the $(\mu\sin\theta+\cos\theta)$ part, the part in $\{\}$ and the denominator.

The denominator is definitely always positive as long as it is non-zero. It is non-zero when $\mu=\frac{\sin\theta}{\cos\theta}$. The part inside the $\{\}$ is also positive.

The only concern is $(\mu\sin\theta+\cos\theta)$. But if $\mu=\frac{\sin\theta}{\cos\theta}$, then $\mu\sin\theta+\cos\theta=\mu^2\cos\theta+\cos\theta=(\mu^2+1)\cos\theta$.

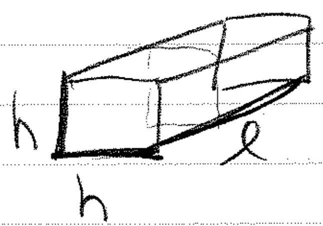
This quantity is positive as long as $0<\theta<\pi/2$.

Since F'' is positive, this means F is concave up and $\mu=\tan\theta$ does minimize F.

(The only possible exception to this argument is at $\theta=0$, $\theta=\pi/2$, i.e. the endpoints. I won't show the details here, but it is true that $F(\tan^{-1}\mu)$ does give a minimum.)

37) (con't. from bottom) So the dimensions of maximum volume would be $r = 36/\pi$ in, $l = 36$ in

15

37) 

Girth = distance around center of box
l = length of box
h = height + width of box

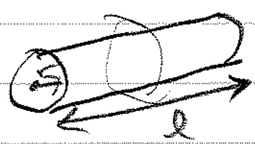
Find $l + h$ that gives maximum volume
Subject to ~~l+~~Girth = 108 in.

Maximize $V = h^2 l$ Constraint: $l + 4h = 108$ in.
$\Rightarrow l = 108\text{ in} - 4h$

$V = h^2(108 - 4h)$
$V(h) = 108h^2 - 4h^3$
Find critical numbers.
$V'(h) = 2 \cdot 108 \cdot h - 12h^2 = 216h - 12h^2$
$V'(h) = 0$
Mathematica gives me: $h = 9$. and $V''(9) = -24$.
Since $V''(9)$ is negative, I know V is concave down and using the 2nd derivative test this means $h = 9$ in gives a maximum volume.

$l = 108\text{ in} - 4(9\text{ in}) = 72\text{ in}.$

$\boxed{l = 72\text{ in. and } h = 9\text{ in yield maximum mailable volume}}$



r = radius of tube
l = length of tube

Maximize $V = \pi r^2 l$ with Constraint $l + 2\pi r = 108$ in
$V(r) = \pi r^2(108 - 2\pi r)$ $\Rightarrow l = 108 - 2\pi r$
From Mathematica: $V'(r) = 0$ yields critical numbers:
$r = 0$, $r = 36/\pi$. $V''(0) = 216\pi > 0$, so V is Concave Up ($r = 0$ yields min Volume)
$V''(36/\pi) = -216\pi < 0$, so V is concave down & $r = 36/\pi$ yields largest volume.
CON'T ABOVE