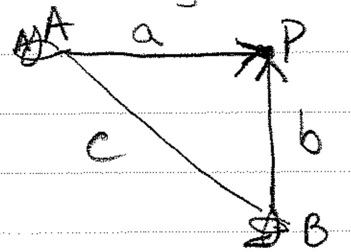
19) (Looking from above:)



a = distance from plane A to point P VAR

b = " " " B " " " VAR

c = distance between planes VAR

Neg b/c of the way a+b are defined

Given $\frac{da}{dt} = -450$ mph, $\frac{db}{dt} = -600$ mph

Find the rate at which c is decreasing when a = 150 mi + b = 200 mi:

$$a^2 + b^2 = c^2$$

$$\frac{d}{dt}(a^2+b^2) = \frac{d}{dt}(c^2)$$

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$\frac{dc}{dt} = \frac{a}{c}\frac{da}{dt} + \frac{b}{c}\frac{db}{dt}.$$

Find c when a = 150 mi, b = 200 mi:

$$c = \pm\sqrt{(150\,mi)^2 + (200\,mi)^2}$$

$$c = + 250\,mi$$

$$\frac{dc}{dt} = \frac{150\,mi}{250\,mi}\left(-450\frac{mi}{hr}\right) + \frac{200\,mi}{250\,mi}\left(-600\frac{mi}{hr}\right) = -270\frac{mi}{hr} + \left(-480\frac{mi}{hr}\right)$$

$$\frac{dc}{dt} = -750\frac{mi}{hr}$$

c is decreasing at a rate of 750 mi/hr

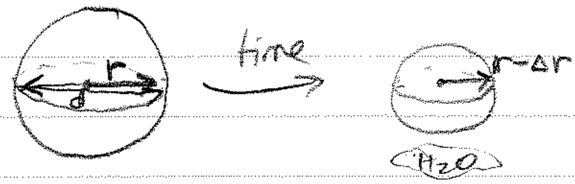
The distance between the planes is 250 mi, but that distance is decreasing at the rate of 750 mi/hr. So clearly I have less than ½ an hour before the planes might crash. In fact, I have:

$$750\,mi/hr * X\,hr = 250\,mi$$

$$X\,hr = \frac{1}{3}\,hr$$

$\frac{1}{3}$ hours to change the course of one of the planes.

22)



r = radius of snowball, VAR
SA = surface area of snowball, VAR
d = diameter of snowball, VAR

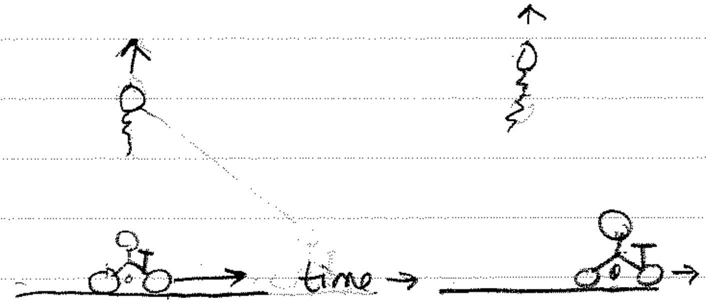
Given $\frac{d\,SA}{dt} = -1\ cm^2/min.$

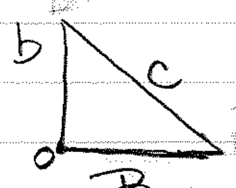
Find rate at which diameter decreases when diameter = 8 cm.

$$SA = 4\pi r^2$$
$$SA = 4\pi\left(\tfrac{1}{2}d\right)^2 = 4\pi\tfrac{1}{4}d^2 = \pi d^2$$
$$\frac{d}{dt}(SA) = \frac{d}{dt}(\pi d^2) = \pi\frac{d}{dt}(d^2)$$
$$\frac{d\,SA}{dt} = \pi\, 2d\,\frac{d\,d}{dt}$$
$$-1\ cm^2/min = 2\pi(8cm)\frac{d\,d}{dt}$$
$$\frac{-1}{16\pi}\frac{cm}{min} = \frac{d\,d}{dt}$$

$\boxed{\text{The diameter decreases at a rate of } -\frac{1}{16\pi}\frac{cm}{min}.}$

23)





b = distance between ground + balloon, VAR
B = distance between O (directly below balloon) and Boy, VAR
c = distance between Boy & balloon.

Given: $\frac{db}{dt} = 5 \text{ ft/s}$, $\frac{dB}{dt} = 10 \text{ ft/s}$, $b(0s) = 30 \text{ ft}$

Find $\frac{dc}{dt}$ after 2 seconds:

After 2 seconds, $b(2) = b(0) + 5 \text{ ft/s} \cdot 2s = 40 \text{ ft}$
$B(2) = 10 \text{ ft/s} \cdot 2 \text{ sec} = 20 \text{ ft}$
(These calculations are valid because the balloon + Boy move at constant speed.)

$$c(2) = \sqrt{(b(2))^2 + (B(2))^2} = \sqrt{(40 \text{ ft})^2 + (20 \text{ ft})^2} = \sqrt{2000 \text{ ft}^2} \doteq 44.7 \text{ ft}$$

$$c^2 = B^2 + b^2$$

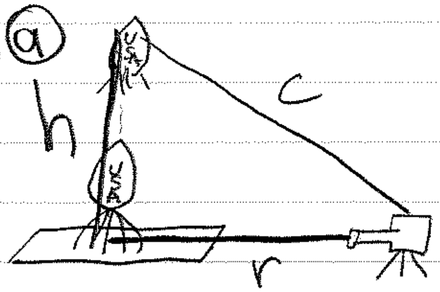
$$\Rightarrow 2c \frac{dc}{dt} = 2a \frac{dB}{dt} + 2b \frac{db}{dt}$$

$$\frac{dc}{dt} = \frac{B}{c}\frac{dB}{dt} + \frac{b}{c}\frac{db}{dt} = \left(\frac{20 \text{ ft}}{44.7 \text{ ft}}\right)\left(10 \frac{\text{ft}}{\text{s}}\right) + \left(\frac{40 \text{ ft}}{44.7 \text{ ft}}\right)\left(5 \text{ ft/s}\right)$$

$$\frac{dc}{dt} = 8.94 \text{ ft/s}$$

The distance between the Boy and the balloon is increasing at a rate of 8.94 ft/s after 2s.

24) a



h = height of rocket, VAR

c = distance between camea + rocket, VAR

r = distance between camera + laundpad, 5 CONST

Given: $\frac{dh}{dt} = 0.2$ mi/s, $r = 5$ mi

Find $\frac{dc}{dt}$ when $h = 3$mi:

$$c^2 = r^2 + h^2$$

$$2c\frac{dc}{dt} = 2r\cancelto{0}{\frac{dr}{dt}} + 2h\frac{dh}{dt}$$

$$\frac{dc}{dt} = \frac{h}{c}\frac{dh}{dt}$$

Find c when h=3mi:

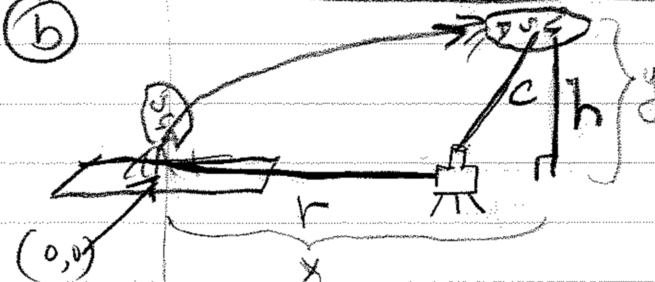
$$c = \sqrt{(5\text{mi})^2 + (3\text{mi})^2} = \sqrt{34\text{mi}^2} \approx 5.83\text{mi}$$

$$\frac{dc}{dt} = \left(\frac{3\text{mi}}{\sqrt{34\text{mi}^2}}\right)\left(0.2\frac{\text{mi}}{\text{s}}\right)$$

$$= 0.102 \text{ mi/s}$$

$\boxed{\text{The rate of change of the distance between the camera + rocket is } 0.102 \text{ mi/s}}$

b



h = height of rocket above earth, VAR

c = distance b/w rocket + camera, VAR

r = distance between camera + laundpad, CONST

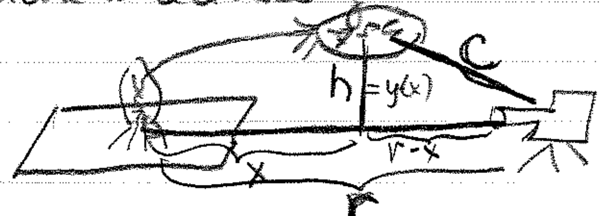
Given $h = 3\sqrt{x}$, $\frac{dh}{dt} = 0.2$ mi/s, $r = 5$mi.

[Note: $h = y(x)$ and x is a function of time]

Find $\frac{dc}{dt}$ when $h = 3$mi:

When $h = 3$mi, $3 = 3\sqrt{x}$ must be true, so $x = 1$ mi.

The picture is actually



NOTE: Part b is quite difficult. You should be able to do part a but don't worry about part b.

10

24) (b) (con't)

So we have $c^2 = h^2 + (r-x)^2$

Let's write x in terms of h:

$h = y(x) = 3\sqrt{\text{mi}}\sqrt{x}$

There has to be units of mi here. (Strange, but true... don't worry about units in this problem ☺)

$h^2 = 9_{\text{mi}} x$

$\frac{1}{9_{\text{mi}}} h^2 = x$

So, $c^2 = h^2 + (r - \frac{1}{9_{\text{mi}}}h^2)^2 = h^2 + r^2 - \frac{2}{9_{\text{mi}}} r h^2 + \frac{1}{81_{\text{mi}^2}} h^4$

Now, taking the derivative of both sides I get:

$$2c \frac{dc}{dt} = 2h \frac{dh}{dt} + \frac{d}{dt}(r^2) - \frac{2}{9_{\text{mi}}} \frac{d}{dt}(rh^2) + \frac{1}{81_{\text{mi}^2}} 4 h^3 \frac{dh}{dt}$$

(remembering that r is a constant,)

$$2c \frac{dc}{dt} = 2h \frac{dh}{dt} + 0 - \frac{2}{9_{\text{mi}}} r\, 2h \frac{dh}{dt} + \frac{4}{81_{\text{mi}^2}} h^3 \frac{dh}{dt}$$

Now,

$$\frac{dc}{dt} = \frac{h}{c}\frac{dh}{dt} - \frac{2}{9}\frac{rh}{c}\frac{dh}{dt} + \frac{2}{81_{\text{mi}^2}}\frac{h^3}{c}\frac{dh}{dt}$$

Find c:

$$\boxed{c = \sqrt{h^2 + (r-x)^2} = \sqrt{(3\text{mi})^2 + (5\text{mi} - 1\text{mi})^2} = \sqrt{9\text{mi}^2 + 16\text{mi}^2} = 5\text{mi}}$$

$$\frac{dc}{dt} = \frac{3\text{mi}}{5\text{mi}}(0.2\,\text{mi/s}) - \frac{2}{9\text{mi}}\frac{(5\text{mi})(3\text{mi})}{5\text{mi}}(0.2\,\text{m/s}) + \frac{2}{81_{\text{mi}^2}}\frac{(3\text{mi})^3}{5\text{mi}}(0.2\,\text{mi/s})$$

$= 0.12\,\text{mi/s} - .1\overline{3}\,\text{mi/s} + 0.02\overline{6}\,\text{mi/s}$

$\approx 0.0133\,\text{mi/s}$

$$\boxed{\text{The rate of change in distance between camera + rocket is now } 0.0133\,\text{mi/s}}$$