Optional Practice
for MA104 WPR I
Spring, 2006

Some Chain Rule practice problems:

1. $y(x) = \tan(\sin x)$
2. $f(t) = \mathbf{e}^{t\cos t}$
3. $y(t) = 2^{\sin(\pi x)}$
4. $f(x) = \sqrt{6x+1}$
5. $f(x) = \mathbf{e}^{5x+1}$
6. $f(t) = \tan(5x^2+3)$
7. $p(\theta) = 3^{\theta^2+1}$
8. $f(x) = \sin(2x)$
9. $g(x) = \mathbf{e}^{-2x^2}$
10. $f(x) = (5x^2+1)^2$
11. $y(t) = \cos\left(\frac{3}{2}\pi t\right)$
12. $g(x) = \dfrac{3x}{\sqrt{x^2+2}}$
13. $f(x) = \mathbf{e}^{\cot(x)}$
14. $f(x) = x\mathbf{e}^{-x^2}$

15. Find the derivative of $f(x) = \dfrac{x}{x^2-1}$ by using the quotient rule. Now, rewrite $f(x)$ as $x\left(x^2-1\right)^{-1}$ and find the derivative using the product and chain rules.

Some Related Rates practice problems:

16. * Two ships A and B are sailing straight away from the point O along routes such that the angle $AOB = 120°$. How fast is the distance between them changing if, at a certain instant, $OA = 8$ mi, $OB = 6$ mi, ship A is sailing at the rate of 20 mi/hr, and ship B at the rate of 30 mi/hr? (Hint: use the law of cosines.)

17. Two boats are racing with constant speed toward a finish marker, boat A sailing from the south at 13 mph and boat B approaching from the east. When equidistant from the marker the boats are 16 miles apart and the distance between them is decreasing at a rate of 17 mph. Which boat will win the race?

18. * Two cars, car A traveling east at 30 mph and car B traveling north at 22.5 mph, are heading toward an intersection I. At what rate is $\angle IAB$ changing at the instant when car A is 300 feet from the intersection and car B is 400 feet from the intersection?

19. You are an air traffic controller and you spot two planes at the same altitude converging on a point as they fly at right angles to each other. One plane is 150 miles from the point and is moving at 450 miles per hour. The other plane is 200 miles from the point and has a speed of 600 miles per hour. At what rate is the distance between the planes decreasing? How much time do you have to get one of the planes on a different flight path? (Assume the planes will not change speed or direction without your approval.)

20. * If A is the area of a circle with radius r and the circle expands as time passes, find dA/dt in terms of dr/dt. Now, suppose there is a water bottle with a hole in the bottom. Water is coming from the hole and the water spreads in a circular pattern. If the radius of the water increases at a constant rate of 2 cm/s, how fast is the area of the spill increasing when the radius is 8 cm?

21. Oil spilled from a ruptured tanker spreds in a circle whose area increases at a constant rate of 6 mi^2/hr. How fast is the radius of the spill increasing when the area is 9 mi^2?

22. If a snowball melts so that the surface area decreases at a rate of 1 cm^2/min, find the rate at which the diameter decreases when the diameter is 8 cm.

23. A balloon is rising straight up at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 10 ft/s. When he passes under the balloon it is 30 ft above him. How fast is the distance between the boy and the balloon increasing 2 seconds later?

24. (Hard) A camera tracks the launch of a spacecraft. The camera is located 5 miles from the launchpad.

    (a) If the spacecraft launches straight upward and travels at a velocity of 0.2 miles per second, at what rate is the distance between the camera and spacecraft changing with the spacecraft is 3 miles above the launchpad?

    (b) Now assume that the spacecraft travels on a trajectory of $y = 3\sqrt{x}$ towards the camera from the launchpad. If the altitude of the spacecraft is 3 miles above the earth and changing at 0.2 miles per second, at what rate is the distance between the camera and the spacecraft now changing?

Some Mean Value Theorem practice problems:

27. * On the New Jersey Turnpike, Ima Fastdriver traveled from Exit 18W (George Washington Bridge, mile marker 113.8) to Exit 1 (Delaware Memorial Bridge, mile marker 1.2). Ms. Fastdriver is concerned greatly about time so she purchased an EZPass to save time at the toll booths. One a particular sunny day in mid-March, the NJ Transit Authority recorded Ms. Fastdriver entering the turnpike at Exit 18W at 1:30pm and exiting the turnpike at Exit 1 at 2:50pm. Use the Mean Value Theorem to determine whether or not the NJ Transit Authority should issue Ms. Fastdriver a citation.

28. At 2:00pm a car's speedometer reads 30 mi/h. At 2:10pm it reads 50 mi/h. Show that at some time between 2:00 and 2:10 the acceleration is exactly 120 mi/h^2.

29. * Don't forget about the graphic representation of the Mean Value Theorem. Look at problem 1 on page 286 in Section 4.3 to be sure you understand this.

30. (Hard) Two runners start a race at the same time and finish in a tie. Prove that at some time during the race they have the same speed. (Hint: Consider $f(t) = g(t) - h(t)$, where g and h are position functions of the two runners.)

Some Mathematica Max/Min practice problems. For each of these functions on a closed interval, use Mathematica to:

- Plot the function and derivative of the function on the same graph.
- Use the `Solve[]` command to determine the critical numbers of the function.
- Determine function values at the critical numbers and endpoints of the interval.
- Classify all critical numbers and endpoints as local maximum, absolute maximum, local minimum, absolute minimum, or neither max nor min.

31. $f(x) = (-2x^3 + 5\sqrt{x} + 4)^2, \qquad 0 \le x \le 2$

32. $f(x) = \dfrac{12x^5 + 15x^4 - 260x^3 - 30x^2 + 720x}{60}, \qquad -6 \le x \le 4$

33. $g(t) = t^{4/5}(t-4)^3, \qquad 2 \le t \le 6$

34. $f(x) = x^3\mathbf{e}^x, \qquad -4 \le x \le 2$

35. * Plot $f(x) = \sin^3 4x^2 + 3x$ for $0 \le x \le \pi/2$, along with its derivative. What happens if you use the `Solve[]` command to find the critical numbers? Why? From the graph, estimate the critical numbers and then use the `FindRoot[]` command to find the critical numbers. Label each as an absolute max or min, a local max or min, or neither a max nor min.

Some Optimization practice problems:

36. An object with weight w is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle θ with the plane, then the magnitude of the force is $F = \dfrac{\mu w}{\mu \sin\theta + \cos\theta}$ where μ is a positive constant called the coefficient of friction and $0 \le \theta \le \pi/2$. Show that F is minimized when $\tan\theta = \mu$.

37. The US Postal Service will accept a box for domestic shipment only if the sum of the length and girth does not exceed 108 in. What dimensions will give a square ended box the largest possible volume? What would be the dimensions of a cylindrical tube of largest volume that could be shipped? (Here girth means the distance around the box perpendicular to the length.)