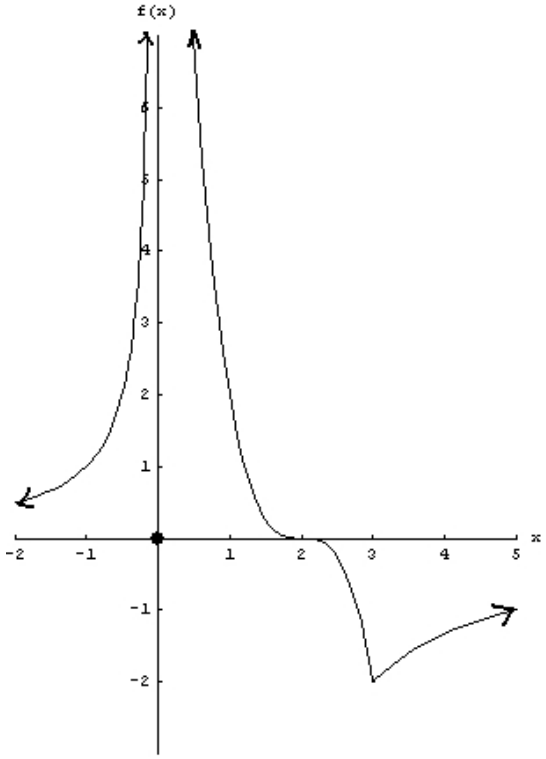
Optional Practice
for MA104 Term End Exam
Spring, 2006

1. Determine the second degree polynomial $f(x) = ax^2 + bx + c$ which has $f''(3) = 6$, $f'(3) = 2$ and $f(3) = -46$.

2. Given the graph of $f(x)$ below, determine the following values. Then, sketch the graph of $f'(x)$.

   a. $\lim_{x \to 0^-} f(x) =$
   b. $\lim_{x \to 0} f(x) =$
   c. $\lim_{x \to 0^+} f(x) =$
   d. $f(0) =$
   e. $\lim_{x \to 3} f(x) =$
   f. $f'(3) =$
   g. $f'(0) =$
   h. $\lim_{x \to 2} f(x) =$



3. Find the following derivatives by hand. Check yourself with Mathematica.

   (a) $f(x) = \mathbf{e}^{(\sin x)(\mathbf{e}^{x^2})}$ (`f[x_] = Exp[Sin[x]*Exp[x^2]]`) Find $f'(x)$.

   (b) $g(x) = \tan\left(\mathbf{e}^{-x^3}\right)$ (`g[x_] = Tan[Exp[-x^3]]`) Find $g'(x)$.

   (c) $h(x, y, z) = 3x^3y + \cos(xz^2) + xy^3z^2$
   (`h[x_,y_,z_] = 3*x^3*y + Cos[x*z^2] + x*y^3*z^2`) Find h_{xz}.

4. In the body, glucose is assumed to be changing continuously into other molecules at a rate proportional to its concentration. This means that the change in the overall concentration of glucose is proportional to the concentration of glucose $g(t)$. If a person does not ingest any glucose (and cannot produce it), glucose will be used at a rate of 4% per hour. Write a differential equation representing this model.

5. Let y be the function which satisfies the differential equation $\frac{dy}{dx} = -3x^2 + 8$ with the initial condition $y(0) = 24$. Use Euler's Method to estimate the value of $y(1)$ by hand using the stepsize $\frac{1}{4}$. Then use Excel or Mathematica to estimate $y(1)$ using the stepsize 0.1.

6. Two unmanned aerial vehicles are flying in the same area. The first UAV is following the trajectory $p1 = \langle t^2, 5t, t^2 - 8t + 20\rangle$ and the second UAV follows trajectory $p2 = \langle 5t^2, 12t, 40 - 25t + 4t^2\rangle$. Use calculus to justify your answers to the following questions.

   (a) During the first 10 seconds, what is the smallest distance between the two UAVs? Do they collide? If so, when?

   (b) During the first 10 seconds, what is the maximum height that each UAV reaches?

7. A human cannonball is to be fired with an initial speed of $v_0 = (80/3)\sqrt{10}$ ft/s. The circus performer hopes to land on a special cushion located 200 ft down range. The circus is being held in a large room with a flat ceiling 75 ft high. Can the performer be fired to the cushion without striking the ceiling? If so, what should the cannon's angle of elevation be?

8. Consider the space curve with parametric equations $x = t, \quad y = t, \quad z = \frac{2}{3}t^{3/2}$. These equations describe the position of an object moving in space. Write a vector or scalar equation of the plane that passes through the point $(1, 1, \frac{2}{3})$ of this curve, and is perpendicular to the tangent of this curve at the same point.

9. Find the dimensions of an unopened soup can of volume 16π cm^3 which has the smallest surface area.

10. You are in charge of erecting a radio telescope on a newly discovered planet. To minimize interference, you want to place it where the magnetic field of the planet is weakest. The planet is perfectly spherical with a radius of 6 units. The strength of the magnetic field is given by $M(x, y, z) = 6x - y^2 + xz + 60$ based on a coordinate system whose origin is at the center of the planet. Where should you locate the radio telescope? (The equation of a sphere centered at (x_0, y_0, z_0) with radius r is $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$.)