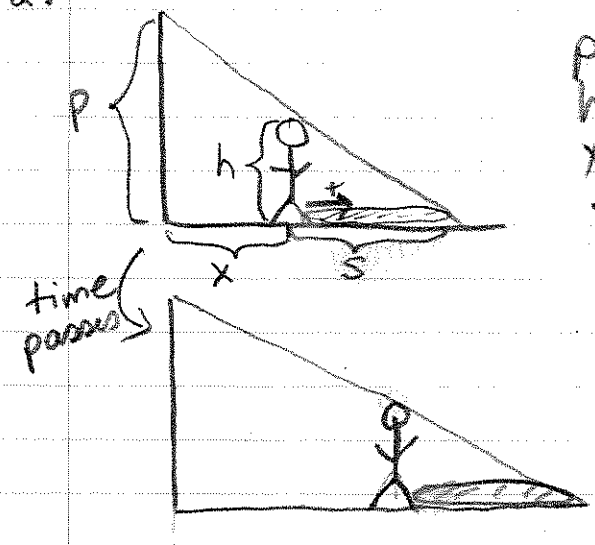
2/21/06

## Section 4.1 Related Rates Practice Problem

(12)




p = height of pole, CONST, 15 ft
h = height of man, CONST, 6 ft
x = dist. b/w man + pole, VAR wrt t
s = length of shadow, VAR wrt t

Given: $p = 15\text{ft}$, $h = 6\text{ft}$, $\frac{dx}{dt} = 5\text{ ft/s}$

Find $\frac{d}{dt}(x+s)$ when $x = 40\text{ft}$.

(Note: $\frac{ds}{dt}$ is the rate at which the shadow's length is changing. $\frac{d}{dt}(x+s)$ is the rate at which the shadow's tip is moving.)

From similar triangles,

$$\frac{s}{h} = \frac{x+s}{p}$$

$$sp = h(x+s)$$

$$= hx + hs$$

So $s = \frac{h}{p-h}x$

Now, $\frac{d}{dt}(x+s) = \frac{d}{dt}\left(x + \frac{h}{p-h}x\right) = \frac{dx}{dt} + \frac{h}{p-h}\frac{dx}{dt}$

$$= 5\frac{\text{ft}}{\text{s}} + \frac{6\text{ft}}{15\text{ft}-6\text{ft}}\,5\frac{\text{ft}}{\text{s}}$$

$$= 5\frac{\text{ft}}{\text{s}} + \frac{6\text{ft}}{9\text{ft}}\,5\frac{\text{ft}}{\text{s}}$$

$$= \frac{25}{3}\frac{\text{ft}}{\text{s}}$$

The tip of the shadow is moving to the right at $\frac{25}{3}\frac{\text{ft}}{\text{s}}$.