Optional Practice
for MA104 Course Quiz II
Spring, 2006

1. Consider the parametric equations $x(t) = \mathbf{e}^t$, $y(t) = \mathbf{e}^{2t} + 1$. Determine the Cartesian equation from this set of parametric equations.

2. Give parametric equations equivalent to the Cartesian equation $y = 4(x-1)^3$.

3. Let the position of one particle be described by $x_1(t) = 2t + 1$, $y_1(t) = 4t^2$ and the position of a second particle be described by $x_2(t) = 3t$, $y_2(t) = 3t$. Do the paths of the two particles ever intersect? Do the particles collide?

4. Find an equation of the largest sphere with center $(6, 5, 7)$ that is contained in the first octant. (See p. 637 for the definition of *first octant*.)

5. What are the lengths of the sides of the triangle PQR with vertices $P(3, -2, -3)$, $Q(7, 0, 1)$, and $R(1, 2, 1)$?

6. You're in the Intergalactic Fire Detection Center (IFDC) (conveniently located at the origin of your galaxy) and think there is an illegal force field in your sector of responsibility. Your team launched three probes and watched them suddenly vaporize at the coordinates $(-10, 10, 3)$, $(10, -10, 3)$, and $(10, 10, 2.5)$. (All coordinates are in Intergalactic Metric Units (IMUs).) Based on knowledge you obtained at the StarFleet Academy, your believe the force field is a plane.

   (a) Find the equation of the plane occupied by the force field.

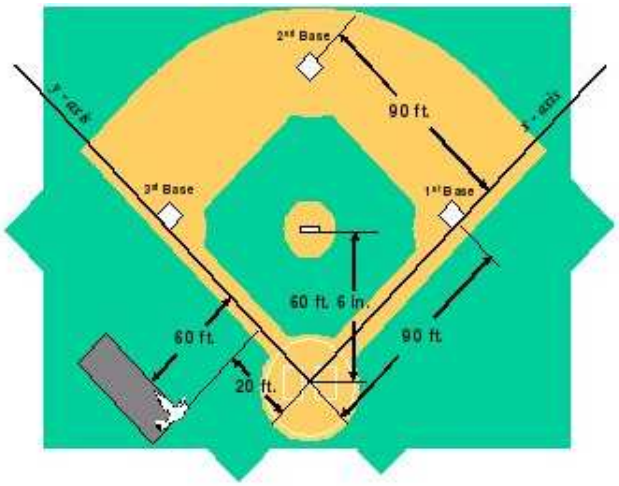
   (b) (Hard) You have a laser is located at $(1, -1, 0)$. You aim the laser at the point on the force field closest to the laser. Find either a vector equation or parametric equation describing the (straight line) path followed by the laser. What direction should you advise the StarTroopers at the IFDC to fire? (What point should you observe to see if the laser hit?)

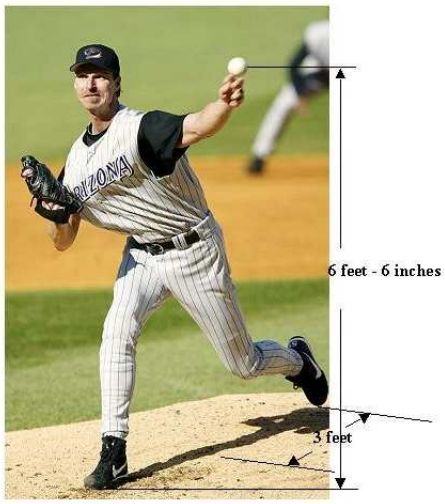
   (c) (Hard) From knowledge obtained at the StarFleet Academy, you know that the force field is weakest where it intersects the xy-plane. Find the parametric equation of the line where the force field intersects the xy-plane.

7. During the Diamondbacks' victory over the Giants on March 24, 2001, pitcher Randy Johnson hit and killed a bird flying in front of home plate. Johnson's pitch to the Giants' Calvin Murray was about three-fourths (45 feet) of the way to home plate when it struck the bird.

Analysis shows that Randy Johnson released the pitch 6 feet, 6 inches above the playing surface and 3 feet in front of the pitching rubber. (The pitching rubber is the white strip which is exactly 60 feet, 6 inches from home plate.) The parametric equation below describes the position of the baseball, measured in feet, as a function of time in seconds. Johnson released the ball when $t = 0$; and the x-axis and y-axis correspond to the first baseline and third baseline, respectively, with the origin at home plate (see figure).

$$\langle -10.2t^2 - 91.5t + 40.6, -9.6t^2 - 91.6t + 40.6, -16.1t^2 + 1.7t + 6.5 \rangle$$





(a) Using the parametric equation provided above, what was the *speed* of the baseball when it hit the dove at $t = 0.32$ seconds?

8. An infantryman reads the directions on the side of a stinger missle and engages and enemy aircraft with this heat seeking missile. (The missile locks onto the target heat source and always moves towards its target.) If he had been thinking of it at the time of the launch, this soldier could have modeled the flight of the enemy aircraft using the vector function $\mathbf{a}(t)$ and the path of the missile using the vector equation $\mathbf{m}(t)$. (Distances are in meters and time in seconds.)

$$\begin{aligned} \mathbf{a}(t) &= \langle 900 + 150t, -450 + 300t, 3690 - 50t \rangle \\ \mathbf{m}(t) &= \langle 250t, 250t, 40t^2 \rangle \end{aligned}$$

(a) Will the missile hit the intended target? If so, at what time?

(b) If successful, how high above the ground is the explosion?

9. Find the scalar and vector projections of $\mathbf{b} = \mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$ onto $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$.

10. Find a vector orthogonal to the plane defined by the points $P(1,0,0)$, $Q(0,2,0)$, $R(0,0,3)$.

11. An incoming target is following a path described by the parametric equations $x(t) = 45t$, $y(t) = 50t - 16t^2 + 6$. You have fired a projectile trying to intercept the incoming target. Your projectile is following a path described by the parametric equations $u(t) = 100 - 40t$, $v(t) = 60t - 16t^2 + 5$.

    (a) Will your projectile collide with the incoming target?

    (b) Will the trajectory of your projectile intersect the trajectory of the incoming target? If so, which will get to the point of intersection first?

    (c) (Hard) Your projectile has a 95% probability of destroying the incoming target if it detonates within 12 feet of the target. Can your projectile destroy the incoming target? When should you detonate the projectile?

12. Find a vector of length $\sqrt{3}$ in the direction of $\langle 5, -6, \sqrt{39} \rangle$.

13. Draw two vectors $\mathbf{a}$ and $\mathbf{b}$ in the xy-plane. Graphically represent $\mathbf{a} + \mathbf{b}$ for the vectors you drew.

14. Draw two non-parallel vectors $\mathbf{a}$ and $\mathbf{b}$ in the xy-plane. Graphically represent $\mathbf{a} - \mathbf{b}$ for the vectors you drew.

15. Find a vector with length 2 which is orthogonal to $\langle 1, 3 \rangle$.

16. Find the projection of $\mathbf{a} = \langle 1, 2 \rangle$ onto $\mathbf{b} = \langle -1, 0 \rangle$ and then represent the projection graphically. What is the component of $\mathbf{a}$ in the $\mathbf{b}$ direction?

17. Let $\mathbf{a} = 2\mathbf{i} - \mathbf{j}$ and $\mathbf{b} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$. Is $\mathbf{a} \times \mathbf{b} \cdot \mathbf{a}$ always zero? Explain. What about $\mathbf{a} \times \mathbf{b} \cdot \mathbf{b}$?

18. Let $\mathbf{a} = 5\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{j} - 5\mathbf{k}$, and $\mathbf{c} = -15\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$. Which pairs are orthogonal? Which pairs are parallel?

19. Vector $\mathbf{a}$ is 4 units long and its direction is $\mathbf{i}$ and vector $\mathbf{b}$ is 6 units long and its direction is $\mathbf{k}$.

    (a) What is the direction of $\mathbf{a} \times \mathbf{b}$? What is the direction of $\mathbf{b} \times \mathbf{a}$?

    (b) What is the magnitude of $\mathbf{a} \times \mathbf{b}$? What is the magnitude of $\mathbf{b} \times \mathbf{a}$?

20. Write a parametric equation through points $(-5, 2, 1)$ parallel to the vector $\langle 1, 1, 6 \rangle$.

21. The position of a projectile relative to the origin is, in meters, $\mathbf{r}(t) = \left\langle 1 + 5t, 2 + 4t + \frac{1}{2}9.8t^2 \right\rangle$. What is the angle between the path of the projectile and the ground at $t = 0.5$?