Problem Set 8
MA104, Spring 2006
DUE: April 12, 2006
Value: 40 points

Instructor: Dr. Leigh Noble
Assigned: April 4, 2006
Recall that this graded assignment must be accompanied by appropriate documentation as per the USMA *Documentation of Written Work*. The assignment is late if not turned in by the beginning of class on the due date. Please justify all answers by showing the important steps in your writeup; be sure to printout your Mathematica worksheet if you use Mathematica for your calculations.

1. Suppose that the directional derivatives of a function $f(x, y)$ at a particular point Q are known in two nonparallel directions. Can you find ∇f at this point? In particular, consider the case in which $D_{\mathbf{u_1}} f(3, 4) = 23/10$ and $D_{\mathbf{u_2}} f(3, 4) = 7/\sqrt{5}$ for $\mathbf{u_1} = \langle 4/5, -3/5 \rangle$ and $\mathbf{u_2} = \left\langle 1/\sqrt{5}, 2/\sqrt{5} \right\rangle$. Try to determine $\nabla f(3, 4)$.

2. Consider the function
$$f(x, y, z) = x^2 + 2y^2 + z^2 - yz$$
and the level surface $f(x, y, z) = 8$.
   (a) Find a vector normal to the level surface at the point $Q(-1, 2, 1)$.
   (b) Find an equation of the plane tangent to the level surface at the same point.

3. Consider the function
$$f(x, y) = \frac{x}{2y + x}.$$
   (a) What is the maximum rate of change of $f(x, y)$ at point $Q(1, 4)$?
   (b) From the same point, in what direction does $f(x, y)$ decrease most rapidly?

4. Three alleles (alternative versions of a gene) A, B, and C determine the four blood types A (AA or AO), B (BB or BO), O (OO) and AB. The Hardy-Weinberg Law states that the proportion of individuals in a population who carry two different alleles is
$$P = 2pq + 2pr + 2rq$$
where p, q, and r are the proportions of A, B and O in the population. Use the fact that $p + q + r = 1$ to show that P is at most $2/3$.