① Sample Solutions 4/3/06

Problem Set 7 – Sample Solutions

1) $z(x,y) = \sqrt{1-x^2} + \ln(3y^2)$

a) Restrictions: $1-x^2 \geq 0$ and $3y^2 > 0$

$1 \geq x^2$ $\qquad y^2 = 0$

$x^2 = 1$ $\qquad$ Sign y^2: $\oplus \underset{0}{|} \oplus$

$\Rightarrow x = \pm 1$ $\qquad$ So $y \neq 0$

$1-x^2$: $\ominus \underset{-1}{\overset{0}{|}} \underset{0}{\oplus} \underset{+1}{\overset{0}{|}} \ominus$

So $-1 \leq x \leq 1$

Ans: $\boxed{D = \{(x,y) : -1 \leq x \leq 1 \text{ and } y \neq 0\}}$

b) $\frac{\partial z}{\partial x} = \frac{1}{2}(1-x^2)^{-1/2}(-2x) + 0$

$\frac{\partial z}{\partial y} = 0 + \frac{1}{3y^2} \cdot 6y$

2) $g(x,y) = \frac{x-5}{2y^2-3} - y^2 + e^{-x^2}$

a) Restrictions: $2y^2 - 3 \neq 0$

$2y^2 - 3 = 0$

$y^2 = 3/2$

$y = \pm\sqrt{3/2}$ $\quad$ So, $y \neq \sqrt{\frac{3}{2}}, y \neq -\sqrt{3/2}$

$\boxed{D = \{(x,y) : y \neq \pm\sqrt{3/2}\}}$

b) $g_x = \frac{1}{2y^2-3} + 0 + e^{-x^2}(-2x) = \frac{1}{2y^2-3} - 2xe^{-x^2}$

$g_{xx} = 0 - 2x(-2xe^{-x^2}) + e^{-x^2}(-2)$

$\boxed{g_{xx} = 4x^2e^{-x^2} - 2e^{-x^2}}$

Problem Set 7 - Sample Solutions (con't)

2) (con't)

$$g_y = (x-5)\left(-(2y^2-3)^{-2}(4y)\right) - 2y + 0$$
$$= -(x-5)\frac{4y}{(2y^2-3)^2} - 2y$$

$$\boxed{g_{yy} = -(x-5)\left\{\frac{(2y^2-3)^2 4 - 4y(2(2y^2-3)(4y))}{(2y^2-3)^4}\right] - 2}$$

3) $h(x,y) = \sin(\pi/3)(x-5)^3 + xy$

a) Domain is all real pairs (x,y)

b) $\frac{\partial h}{\partial x} = \sin(\pi/3)\,3(x-5)^2 + y$

$\frac{\partial h}{\partial y} = x$

c)
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h[x_,y_]= Sin[Pi/3]*(x-3)^3 + x*y
myhx = D[h[x,y],x]
myhy = D[h[x,y],y]
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d) $h_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial h}{\partial x}\right) = \frac{\partial}{\partial y}\left(\sin(\pi/3)3(x-5)^2+y\right) = 1$

$h_{yx} = \frac{\partial}{\partial x}\left(\frac{\partial h}{\partial y}\right) = \frac{\partial}{\partial x}(x) = 1$

e)
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myhxy = D[h[x,y],x,y]

myhyx = D[h[x,y],y,x]
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4) $\frac{\partial f}{\partial x} = f_x = D_1 f = D_x f$

5) Answers will vary, but will follow idea that $f_x(a,b)$ is the slope of the tangent line at point (a,b) in the direction of $\langle 1, 0\rangle$.