2/15/06

Problem Set 4 Sample Solution

1) $f(x)=\dfrac{x-3}{x^2+2}$, $f'(x)=\dfrac{(x^2+2)\frac{d}{dx}(x-3)-(x-3)\frac{d}{dx}(x^2+2)}{(x^2+2)^2}$

$$=\frac{(x^2+2)(1)-(x-3)(2x)}{(x^2+2)^2}$$

$$=\frac{x^2+2-2x^2+6x}{(x^2+2)^2}=\frac{-x^2+6x+2}{(x^2+2)^2}$$

$g(x)=x^3e^x$, $g'(x)=x^3e^x+3x^2e^x$

$h(x)=f(x)g(x)$, $h'(x)=f(x)g'(x)+f'(x)g(x)$

$$=\left(\frac{x-3}{x^2+2}\right)(x^3e^x+3x^2e^x)+\left(\frac{-x^2+6x+2}{(x^2+2)^2}\right)(x^3e^x)$$

$$\boxed{h'(x)=\left(\frac{x-3}{x^2+2}\right)(x^3e^x+3x^2e^x)+\left(\frac{-x^2+6x+2}{(x^2+2)^2}\right)(x^3e^x)}$$

$$h'(2)=\left(\frac{2-3}{4+2}\right)(8e^2+3(4)e^2)+\left(\frac{-4+12+2}{(4+2)^2}\right)(8e^2)$$

$$=\frac{-1}{6}(20e^2)+\frac{10}{36}(8e^2)=\frac{-20e^2}{6}+\frac{80e^2}{36}$$

$$=\frac{-120e^2+80e^2}{36}=\frac{-40e^2}{36}=\boxed{\frac{-10e^2}{9}\approx-8.21}$$

2) $f'(x)=-\csc^2(3\pi+x)\cdot\frac{d}{dx}(3\pi+x)$

$\boxed{f'(x)=-\csc^2(3\pi+x)}$

3) $f'(x)=\dfrac{(\sqrt{x^2+1})(2x)-(x^2-1)\left(\frac{1}{2}(x^2-1)^{-1/2}\cdot(2x)\right)}{x^2+1}$

MJ forgot this.

MJ. forgot to use the chain rule when taking the derivative of $\sqrt{x^2+1}$.

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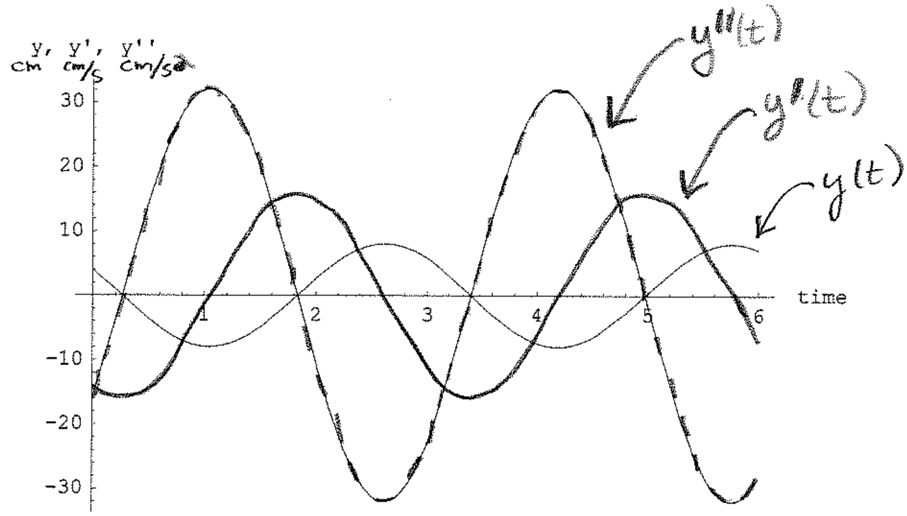
In[1]:= y[t_] := 8*Cos[2*t + Pi / 3]

In[3]:= Plot[{y[t], y'[t], y''[t]}, {t, 0, 6},
         PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 1, 0], RGBColor[0, 0, 1]},
         AxesLabel → {"time", "y, y', y''"}];
```

b)



a) $y(t) = 8\cos(2t + \pi/3)$

$\text{velocity}(t) = y'(t) = -8\sin(2t+\pi/3)\cdot(2) = \underline{\underline{-16\sin(2t+\pi/3)}}$

$\text{acceleration}(t) = y''(t) = -16\cos(2t+\pi/3)(2) = \underline{\underline{-32\cos(2t+\pi/3)}}$

c) $y(6) = 8\cos(12+\pi/3) \simeq \underline{\underline{7.0929 \text{ cm}}}$

$\text{velocity}(6) = y'(6) = -16\sin(12+\pi/3) \simeq \underline{\underline{-7.4002 \text{ cm/s}}}$

$\text{acceleration}(6) = y''(6) = -32\cos(12+\pi/3) \simeq \underline{\underline{-28.3716 \text{ cm/s}^2}}$

The velocity is negative at $t=6$ so the mass is moving downward. In fact, since the position is positive, the mass is moving downward, coming from the apex and moving towards the equilibrium mark.

The acceleration (derivative of velocity) is negative. Hence the velocity is decreasing. The mass is slowing down.