

Problem Set 4
MA104, Spring 2006
DUE: February 15, 2006
Value: 40 points

Instructor: Dr. Leigh Noble, Assigned: February 7, 2006

Recall that this graded assignment must be accompanied by appropriate documentation as per the USMA *Documentation of Written Work*. The assignment is considered late if not turned in by the beginning of class on the due date.

1. (10 points) Consider two functions $f(x)$ and $g(x)$ with $f(x) = \frac{x-3}{x^2+2}$ and $g(x) = x^3e^x$, where e^x is the natural exponential function. Define a third function $h(x)$ to be $h(x) = f(x)g(x)$. Find a formula for $h'(x)$ and use that to find $h'(2)$.
2. (5 points) Differentiate $f(x) = \cot(3\pi + x) + 5$.
3. (10 points) M.J. gave this solution for the derivative of $f(x) = \frac{x^2-1}{\sqrt{x^2+1}}$.

$$f'(x) = \frac{\sqrt{x^2+1}(2x) - (x^2-1)\left(\frac{1}{2}(x^2+1)^{-\frac{1}{2}}\right)}{x^2+1}$$

Explain what M.J. did wrong and show a correct calculation. Be specific— what was the error and what should M.J. have done instead?

4. (15 points) An elastic band is hung from a hook and a mass is hung on the lower end of the band. The mass is pulled downward and then released. Later, an observer walks into the room and sees the mass vibrating vertically. Mesmerized by the fact that there appears to be no damping of motion in this universe (!), the observer determines that the equation $y(t) = 8 \cos(2t + \pi/3)$ represents the motion of the mass t seconds after the observer began recording the motion. A positive position represents a position in centimeters above the resting place of the mass and a negative position is a position below the resting place of the mass.
 - (a) Find equations for the velocity and acceleration at time t .
 - (b) Graph each equation for the time interval $t = 0$ to $t = 6$. Use Mathematica (print your graph and the function used to graph it) or graph by hand. Label the x-axis, y-axis, and whatever else would be needed to assist someone in interpreting your graph.
 - (c) Find the position, velocity, and acceleration at time $t = 6$. In what direction is the mass moving at that time? Is it speeding up or slowing down?