Sample Solution
2/6/06

Problem Set 3, MA104 Sections B3, C3, D3

1) The derivative of $f(x)$ is defined to be

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

For $f(x) = \frac{x}{x-3}$,

$$f'(x) = \lim_{h \to 0} \frac{\frac{x+h}{(x+h)-3} - \frac{x}{x-3}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x+h}{(x+h)-3} - \frac{x}{x-3}}{h} \cdot \frac{((x+h)-3)(x-3)}{((x+h)-3)(x-3)}$$

[multiply by common denominator]

$$= \lim_{h \to 0} \frac{(x+h)(x-3) - x((x+h)-3)}{h((x+h)-3)(x-3)}$$

$$= \lim_{h \to 0} \frac{x^2 - 3x + hx - 3h - x^2 - xh + 3x}{h((x+h)-3)(x-3)}$$

$$= \lim_{h \to 0} \frac{-3h}{h((x+h)-3)(x-3)}$$

[cancel out the h on top & bottom]

$$= \lim_{h \to 0} \frac{-3}{(x+h-3)(x-3)}$$

$$= \frac{-3}{(x-3)(x-3)}$$

so $\boxed{f'(x) = \frac{-3}{(x-3)^2}}$

2) (a) $f'(3)$ does not exist. This is clear if we look at $f(x)$ which is not defined at $x=3$. $f(x)$ will not have a derivative where it is not defined or not continuous. $f(x)$ is also not continuous at $x=3$. We also notice that $f'(3)$ is not defined.

(b) $f'(0)$ is found by substituting $x=0$ into the formula for $f'(x)$ found in part (i).

$$f'(0) = \frac{-3}{(0-3)^2} = \frac{-3}{9} = \boxed{\frac{-1}{3}}$$

3) (a) Differentiate $y = \frac{2x^2 - 3\sqrt{x}}{x}$ by stating each derivative rule used.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2x^2-3\sqrt{x}}{x}\right) = \frac{d}{dx}\left(2x - 3x^{-\frac{1}{2}}\right)$$ [Using algebra to rewrite problem.]

$$= \frac{d}{dx}(2x) - \frac{d}{dx}(3x^{-\frac{1}{2}})$$ Difference Rule

$$= 2\frac{d}{dx}(x) - 3\frac{d}{dx}(x^{-\frac{1}{2}})$$ Constant Multiple Rule

$$= 2(1)x^{1-1} - 3(-\tfrac{1}{2})x^{-\frac{1}{2}-\frac{2}{2}}$$ Power Rule

$$= 2x^0 + \frac{3}{2}x^{-\frac{3}{2}}$$

$$= \boxed{2 + \frac{3}{2}x^{-\frac{3}{2}}}$$

(b) Differentiate $y = \frac{4}{t^5} + 5\pi t$ by stating each rule used.

$$\frac{dy}{dt} = \frac{d}{dt}\left(\frac{4}{t^5} + 5\pi t\right) = \frac{d}{dt}(4t^{-5} + 5\pi t)$$ [Using algebra to rewrite problem.]

$$= \frac{d}{dt}(4t^{-5}) + \frac{d}{dx}(5\pi t)$$ Sum Rule

$$= 4\frac{d}{dt}(t^{-5}) + 5\pi\frac{d}{dt}(t)$$ Constant Multiple Rule

$$= 4(-5)t^{-5-1} + 5\pi(1)t^{1-1}$$ Power Rule

$$= \boxed{-20t^{-6} + 5\pi}$$