Problem Set 2
MA104, Spring 2006
DUE: January 27, 2006
Value: 40 points

Instructor: Dr. Leigh Noble

Assigned: January 20, 2006

Recall that this graded assignment must be accompanied by appropriate documentation as per the USMA *Documentation of Written Work*. The assignment is considered late if not turned in by the beginning of class on the due date.

1. Consider a function y which has the rate of change $y' = 2xy^2$ and initial condition $y(0) = 1$. Use Euler's Method to approximate the value of $y(0.4)$ using these various stepsizes: $h = 0.1$, $h = 0.05$, $h = 0.02$, $h = 0.01$.

   How confident are you in your estimation of $y(0.4)$ using the stepsize $h = 0.1$ as compared with the value estimated using stepsize $h = 0.01$? What is a benefit of reducing the stepsize? What are some practical reasons which might prevent you from reducing the stepsize even further (to some incredibly small number)?

   For this problem you will need the help of a computer to determine the estimated values for each step size. I do not need you to turn in your computer program/worksheet. What I want is the estimated values for each stepsize along with the first three steps of the Euler's method written out by hand for each stepsize. Also, answer the discussion questions using a few sentences or a paragraph or two.

2. If a ball is thrown into the air with a velocity of 42 ft/s, its height in feet t seconds later is given by $y(t) = 5 + 42t - 16t^2$.

   (a) Find the average velocity for the time period beginning when $t = 2$ and lasting (i) 1/2 second, (ii) 1/10 second, and (iii) 1/100 second.

   (b) Estimate the instantaneous velocity when $t = 2$.

3. Sketch (by hand) the graph of an example of a single function f that satisfies all of the following conditions:

   $\lim_{x\to-\infty} f(x) = -2 \quad \lim_{x\to\infty} f(x) = 0 \quad \lim_{x\to-3} f(x) = \infty$

   $\lim_{x\to3^-} f(x) = -\infty \quad \lim_{x\to3^+} f(x) = 2$

   f is continuous from the right at 3.