

1/19/06  
MA104Problem Set 1

- 1)  $P$  = total population (constant)  
 $r$  = number of people who know the rumor,  $r(t)$   
 $u$  = number of people who don't know the rumor,  $u(t)$

$$\frac{dr}{dt} = kr u \quad , \quad \left[ \frac{dr(t)}{dt} = k r(t) u(t) \right]$$

where  $k$  is a proportionality constant (positive)  
 and  $\frac{dr}{dt}$  is the rate that the rumor is spreading.

Note:  $r + u = P$ , so we can rewrite the equation as  $\frac{dr}{dt} = k r (P - r)$

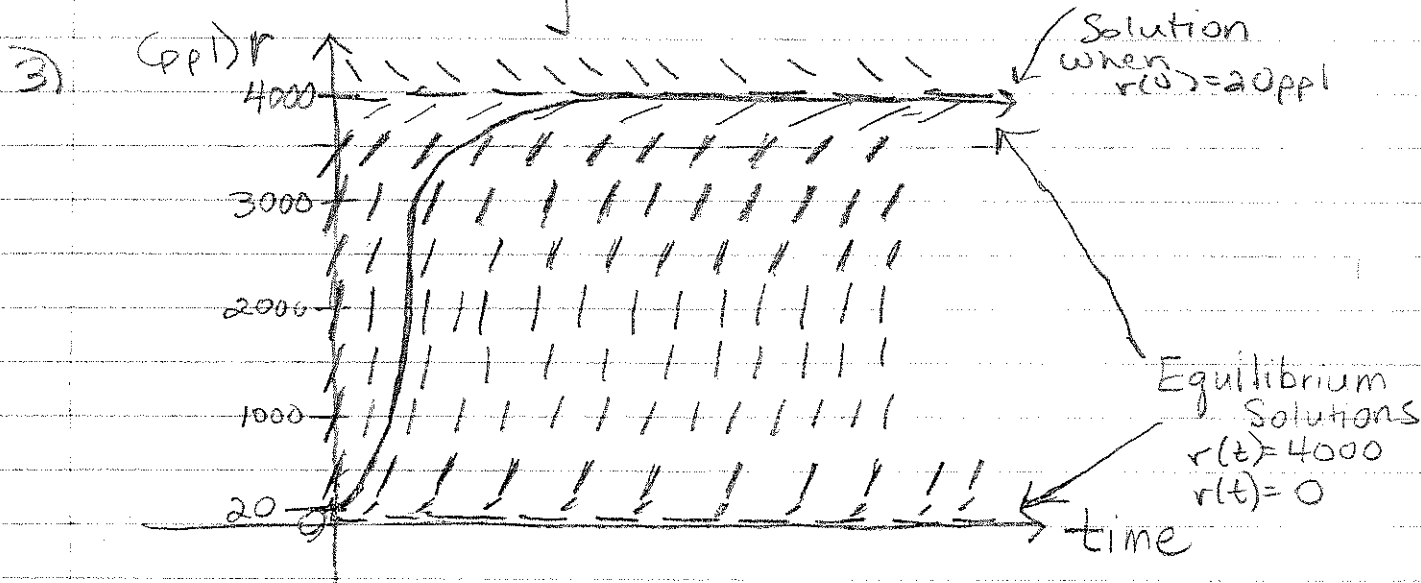
- 2) Answers will vary. My thoughts:  
 This model seems a reasonable first approximation for the rate of spread of a rumor. If few people know a rumor, it spreads slowly at first. It gains momentum and spreads more rapidly as more people learn of it but when half the population knows it, it begins to spread more slowly until (almost) the entire population knows it. If many people know the rumor initially, it spreads very rapidly until half the population knows. Then it follows the same behavior as described above.

However, this model does not take into account the complexity of human behavior. Some people may never come into contact with persons who know the rumor and could never receive it. Some people may never spread a rumor which could reduce the transmission rate. The rumor could be garbled or altered during transmission which would →

2) (cont)

mean there were now two rumors spreading, which this model does not take into consideration. My final thought is that this model does not take into consideration the case when the transmission rate might jump such as when an entire auditorium full of people hear a rumor.

This model is a reasonable first approximation but there are many refinements that could be made.



Notes: Slopes on each horizontal line are the same.

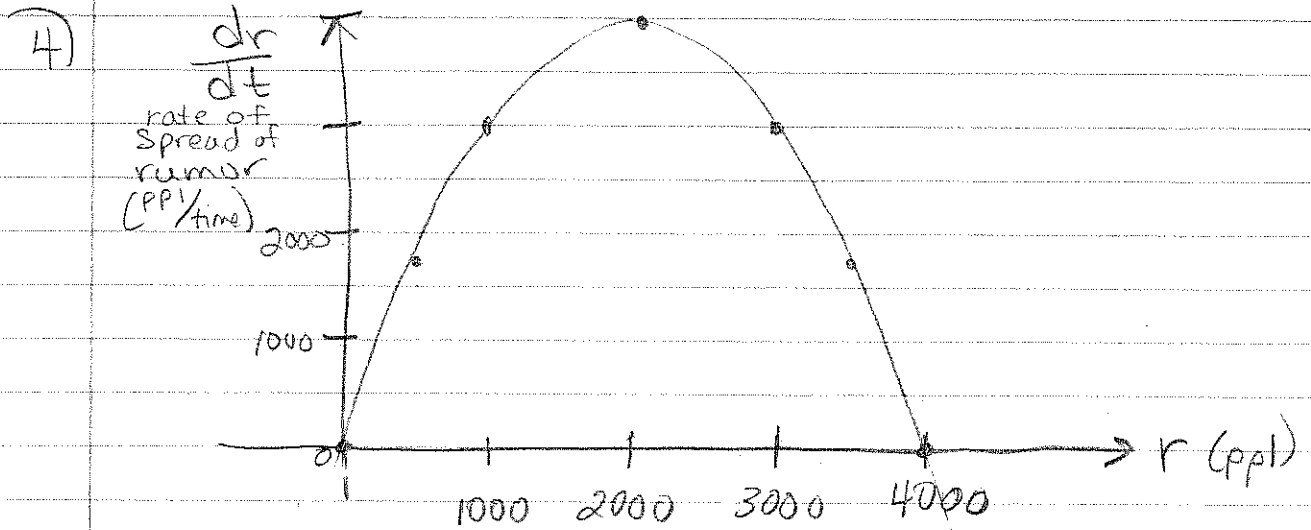
$$\text{Slope at } (0, 500) = \text{Slope at } (0, 3500) = 1750$$

$$\text{Slope at } (0, 1000) = \text{Slope at } (0, 3000) = 3000$$

$$\text{Slope at } (0, 1500) = \text{Slope at } (0, 2500) = 3750$$

$$\text{Slope at } (0, 2000) = 4000$$

$$\text{Slope at } (0, 4500) = -2250$$



The entire population will know the rumor. This can be seen by the graph in part 3. The graph here in part 4 shows that when 2000 people know the rumor it is spreading as rapidly as it can.