Problem Solving Lab, Lesson 46
Board Problems

1. Find the gradient of the function $f(x, y) = x^2 + y^2$ at the point $(1, 2)$. Then find the directional derivative at that point in the direction of the vector $2\mathbf{i} - 3\mathbf{j}$.

2. Find the gradient of the function $f(x, y, z) = x^2 + y^2 + z^2$ at the point $(2, 1, 3)$. Then find the directional derivative from that point in the direction of the origin.

3. Find the gradient of the function $f(x, y) = (x - y)^3 + x^4$ at the point $(3, 1)$. Then find the directional derivative from that point in the direction of point $(-2, 1)$.

4. Find the directional derivative of the function $f(x, y, z) = \dfrac{2x}{y + z}$ at the point $P(4, 1, 1)$ in the direction of $\mathbf{v} =< 1, 2, 3 >$.

5. Find the indicated partial derivative:

$$f(x, y, z) = x^5 + 3x^4y^4z^3 + yz^2, \qquad f_{xyz}$$

6. Let $V(r, h)$ represent the volume of a cylinder where r is the radius of the cylinder and h is its height. Calculate $\dfrac{\partial V}{\partial r}$ and explain the meaning.

7. A surface is described by the equation $f(x, y) = 5xy - 2y^2$. Find the gradient of f. At the point $(1, 3)$, in what direction is the rate of change of the surface elevation the greatest? Why?

8. Find a sketch the domain of the function $f(x, y) = \sqrt{x^2 + y^2 - 1} + \ln(4 - x^2 - y^2)$. (Try to use *set notation*.)

9. Find all the directions in which the directional derivative of $f(x, y) = x^2y^2 + 3x - 2$ at the point $(3, -1)$ has the value 0.

10. Section 11.6 Problems # 30, 34, 27

11. Try to sketch several level curves of the function $f(x, y) = 2x - y^2$. Then use Mathematica to check yourself. Mark the point $(2, 2)$ and calculate the gradient at that point. Draw the gradient on your graph. What do you notice about the relationship between the gradient vector and the level curves?

12. Calculate the first partial derivatives for $g(r, s, t) = \ln(3r^2s) + \cos(5t) - \sin(3rt)$ at the point $(1, 2, 2/10)$.