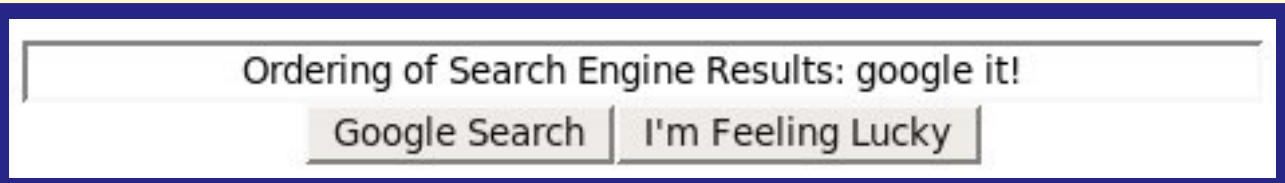
Ordering of Search Engine Results

google it!

Leigh L. Noble, PhD
noble@leighnoble.org

Spuyten Duyvil Undergraduate Mathematics Conference

Some Questions



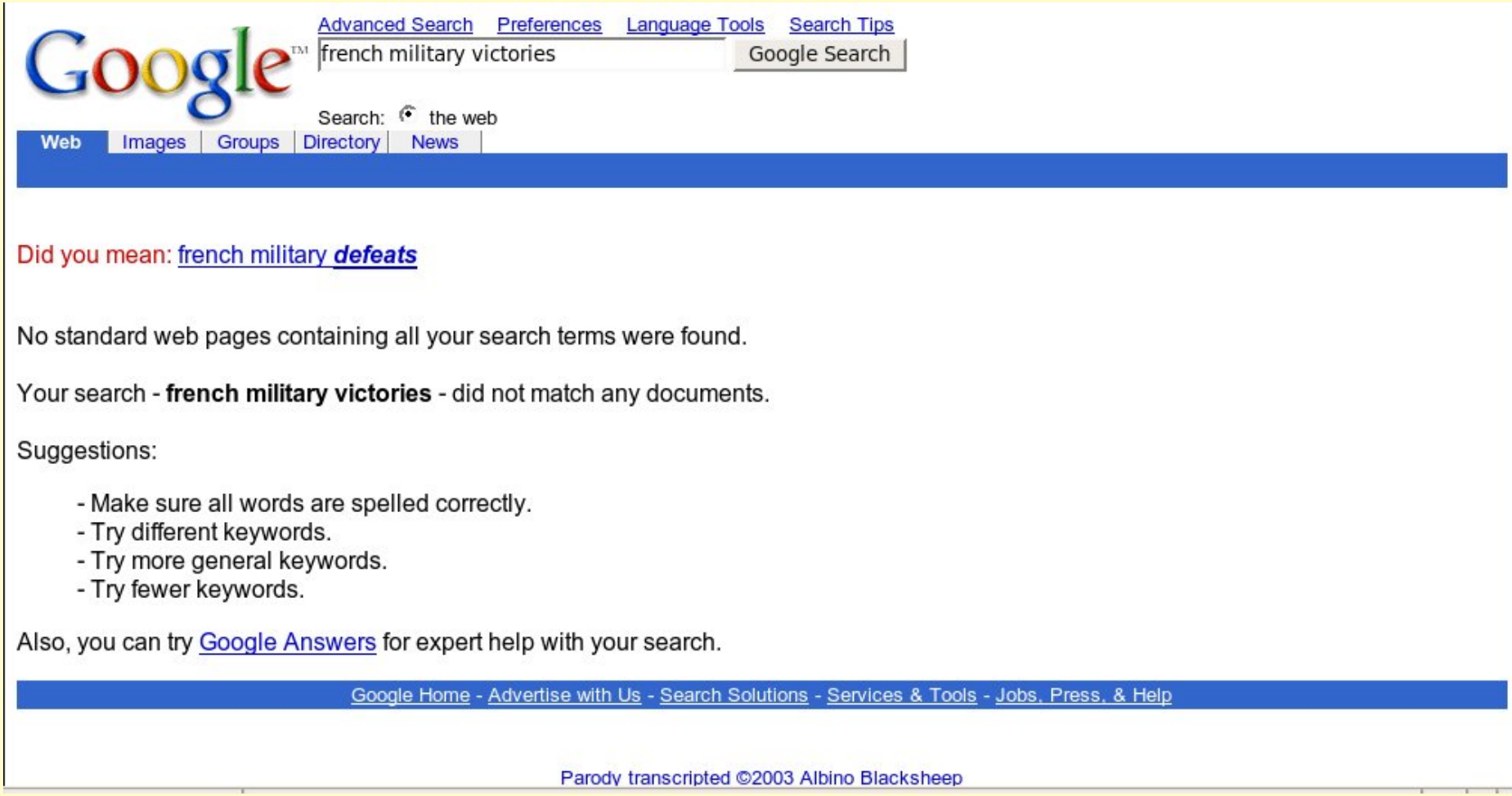
- What's a *Google bomb*?
- How does Google order the results of your search?
- Does "mutual linking" improve a site's position?

MATH: Use linear algebra to calculate importance scores

- Nice mini-web example
- Dangling node
- Disconnected web
- The Google matrix

Search for **french military victories** I'm Feeling Lucky

http://www.albinoblacksheep.com/text/victories.html



Google™ Advanced Search Preferences Language Tools Search Tips

french military victories Google Search

Search: the web

Web Images Groups Directory News

Did you mean: french military ***defeats***

No standard web pages containing all your search terms were found.

Your search - **french military victories** - did not match any documents.

Suggestions:

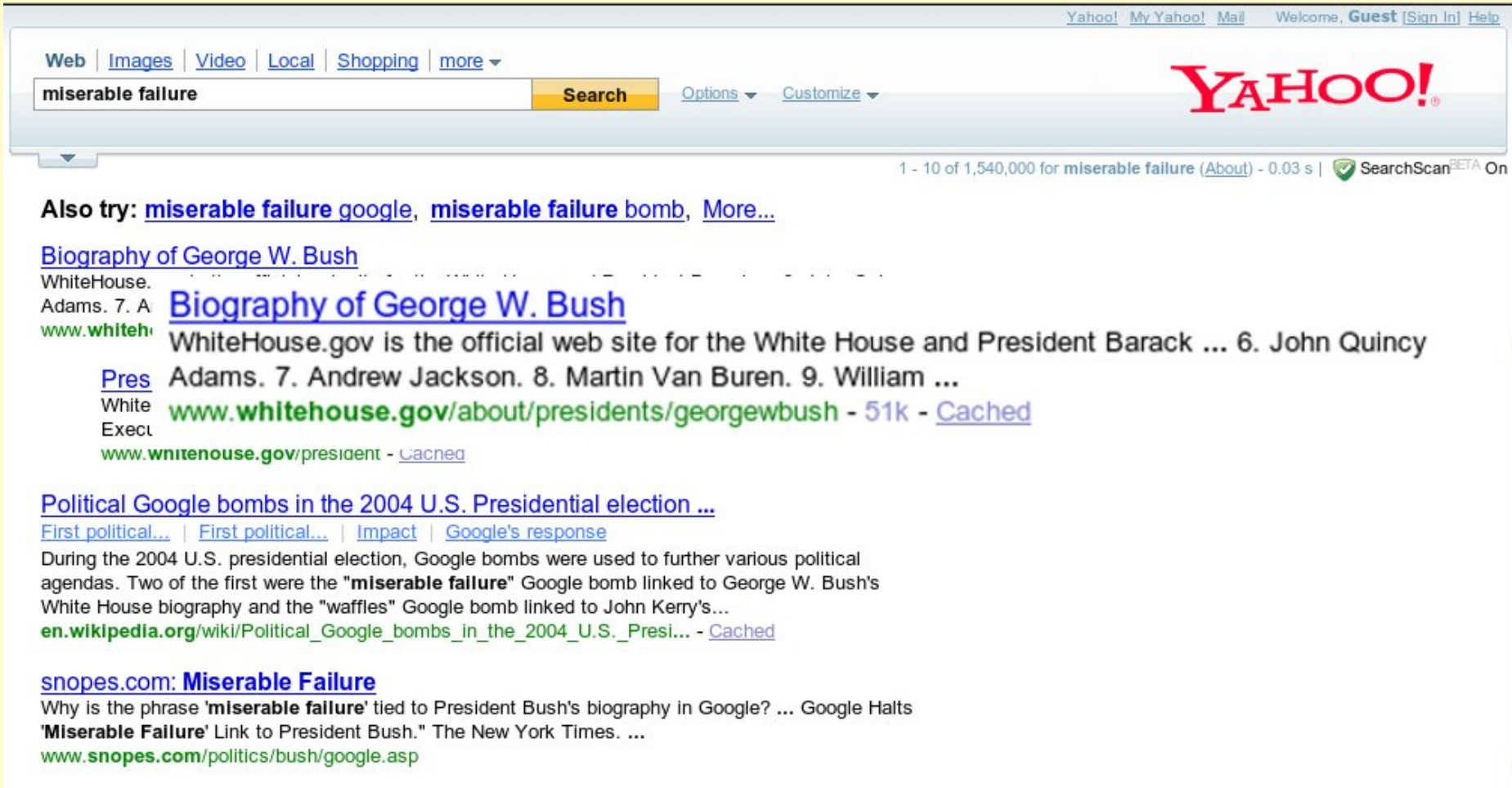
- Make sure all words are spelled correctly.
- Try different keywords.
- Try more general keywords.
- Try fewer keywords.

Also, you can try Google Answers for expert help with your search.

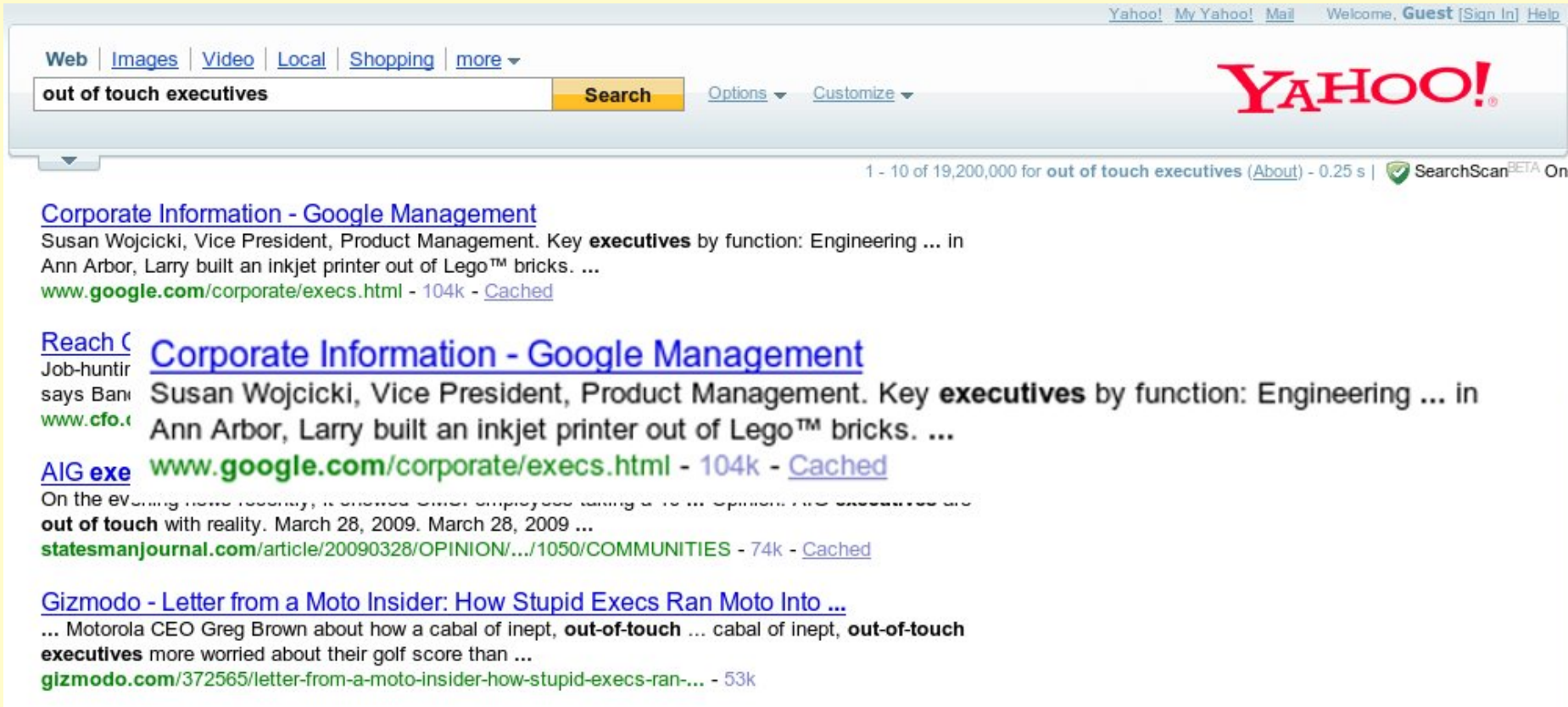
Google Home - Advertise with Us - Search Solutions - Services & Tools - Jobs, Press, & Help

Parody transcripted ©2003 Albino Blacksheep

Search for *miserable failure*



Search for **out of touch executives**



"Link Bomb" is more descriptive

Works off anchor text:

```
Click here to read about a <a href =
"http://www.whitehouse.gov/about/presidents/georgewbush">
miserable failure</a>.
```

Creates this text on a web page:

Blah blah blah, and more blah. Click here to read about a miserable failure. And now for something completely different...

The phrase *miserable failure* is now associated with the bomb recipient's website.

Big Picture

1. Crawl / Webspider
2. Index the data the spider gathers
3. Search and Presentation of User Interface

Historical Perspective

→ Exact IP address and file name/location

1990 → Archie: only searched titles on per-server basis

→ Veronica: fancier version of Archie

→ WebCrawler: full text search

1994 → Lycos: used anchor text to rate relevance

→ Yahoo: directory, no search

1995 → AltaVista: indexed 10 million docs in 1995

→ Excite: grouped pages by keyword

1996 → Google: BackRub+**PageRank**, later added full-text search

* FACTOID: From 1993 through 1996 the WWW grew from 130 sites to $>$ 600,000

Importance Scores

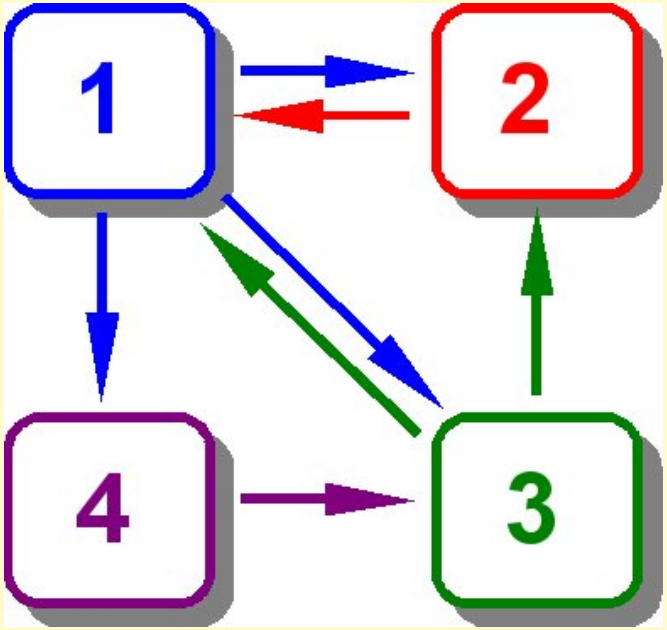
Idea behind *PageRank* algorithm:
Each page lends part of its importance to the pages it links to.

$$\mathbf{R}(p_k) = \sum_{p_j \in \mathbb{B}_k} \frac{\mathbf{R}(p_j)}{n_j}$$

$\mathbb{B}_k$ = all pages linking to p_k (aka *backlinks*)

n_j = the number of links going OUT of page p_j

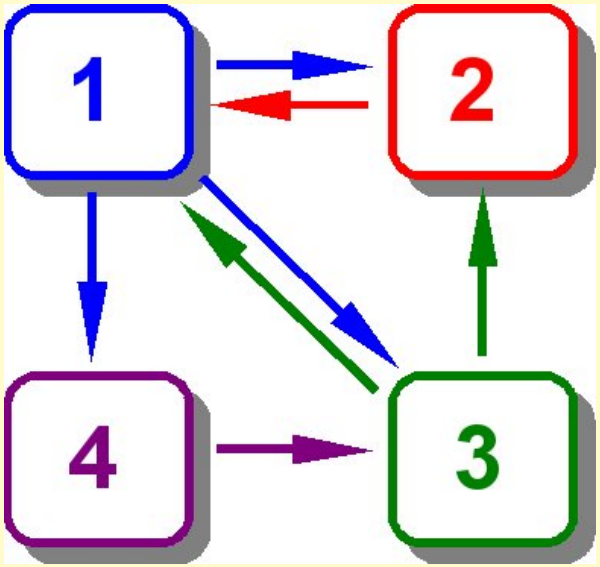
Example 1: Best Case



$$\mathbf{R}(p_k) = \sum_{p_j \in \mathbb{B}_k} \frac{\mathbf{R}(p_j)}{n_j}$$

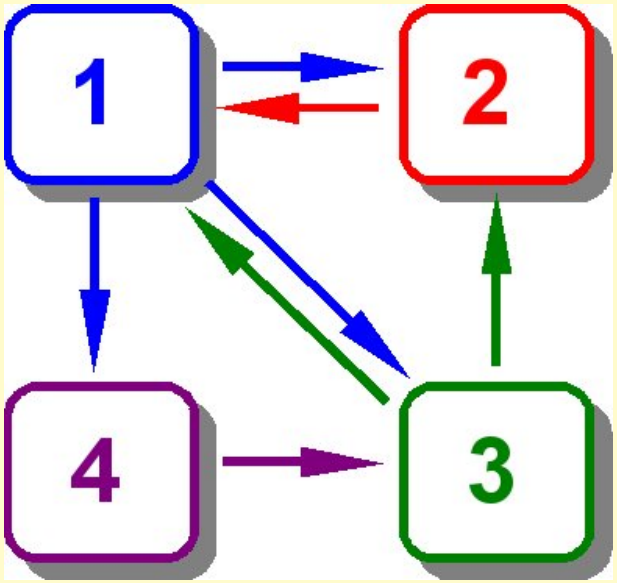
$$\begin{aligned}
\mathbf{R}(p_1) &= 0 \cdot \mathbf{R}(p_1) + 1 \cdot \mathbf{R}(p_2) + 1/2 \cdot \mathbf{R}(p_3) + 0 \cdot \mathbf{R}(p_4) \\
\mathbf{R}(p_2) &= 1/3 \cdot \mathbf{R}(p_1) + 0 \cdot \mathbf{R}(p_2) + 1/2 \cdot \mathbf{R}(p_3) + 0 \cdot \mathbf{R}(p_4) \\
\mathbf{R}(p_3) &= 1/3 \cdot \mathbf{R}(p_1) + 0 \cdot \mathbf{R}(p_2) + 0 \cdot \mathbf{R}(p_3) + 1 \cdot \mathbf{R}(p_4) \\
\mathbf{R}(p_4) &= 1/3 \cdot \mathbf{R}(p_1) + 0 \cdot \mathbf{R}(p_2) + 0 \cdot \mathbf{R}(p_3) + 0 \cdot \mathbf{R}(p_4)
\end{aligned}$$

$$\begin{bmatrix} 0 & 1 & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & 0 & 1 \\ \frac{1}{3} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{R}(p_1) \\ \mathbf{R}(p_2) \\ \mathbf{R}(p_3) \\ \mathbf{R}(p_4) \end{bmatrix} = \begin{bmatrix} \mathbf{R}(p_1) \\ \mathbf{R}(p_2) \\ \mathbf{R}(p_3) \\ \mathbf{R}(p_4) \end{bmatrix}$$



Ways to solve:

- Like a traditional eigenvector problem with eigenvalue $\lambda = 1$
- Or find a stationary vector so that $\mathbf{P}^m\mathbf{R} = \mathbf{R}$.

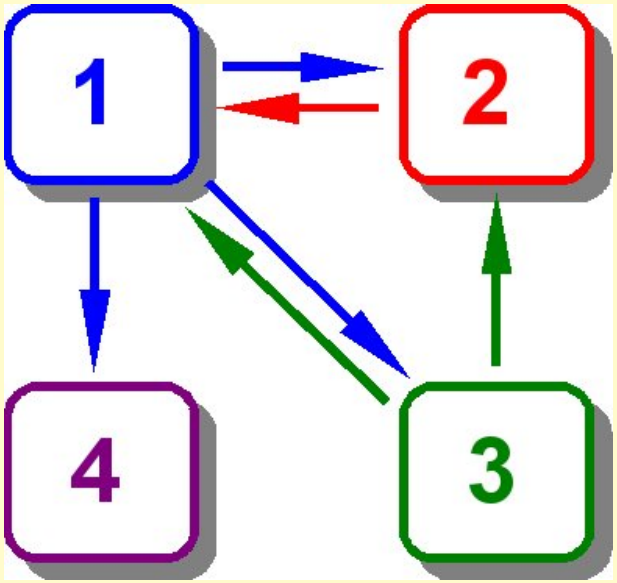


Normalize column sum to 1:

$$\begin{bmatrix} \mathbf{R}(p_1) \\ \mathbf{R}(p_2) \\ \mathbf{R}(p_3) \\ \mathbf{R}(p_4) \end{bmatrix} = c \begin{bmatrix} 3 \\ 2 \\ 2 \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 0.375 \\ 0.250 \\ 0.250 \\ 0.125 \end{bmatrix}$$

Page 1 has the highest importance score!

Example 2: Dangling Node



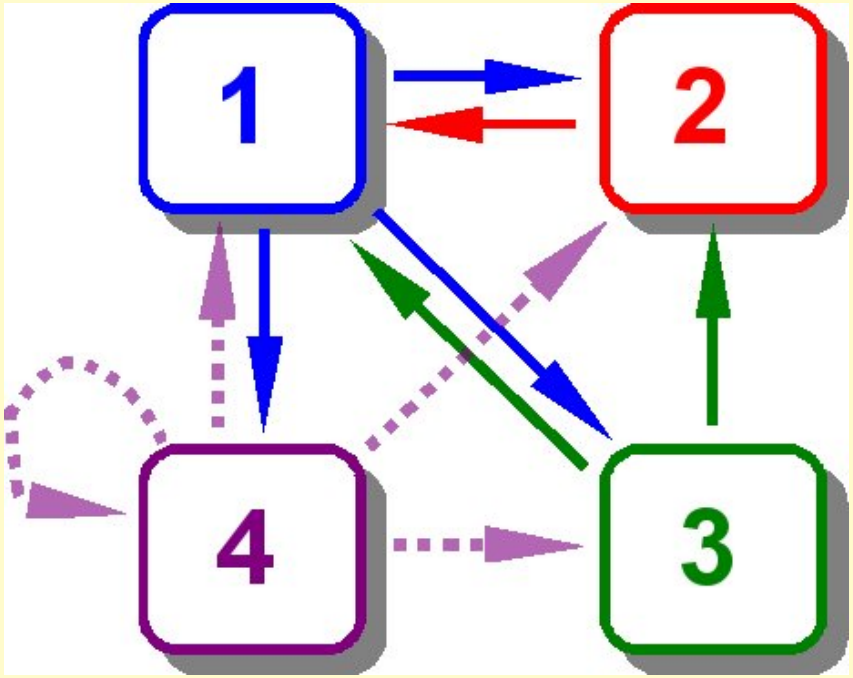
$$\mathbf{R}(p_k) = \sum_{p_j \in \mathbb{B}_k} \frac{\mathbf{R}(p_j)}{n_j}$$

$$\begin{bmatrix} 0 & 1 & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{R}(p_1) \\ \mathbf{R}(p_2) \\ \mathbf{R}(p_3) \\ \mathbf{R}(p_4) \end{bmatrix} = \begin{bmatrix} \mathbf{R}(p_1) \\ \mathbf{R}(p_2) \\ \mathbf{R}(p_3) \\ \mathbf{R}(p_4) \end{bmatrix}$$

Has no eigenvalue of 1!

$$\lambda \in \{0,\ 0.836,\ -0.418 \pm 0.156i\}$$

Random Surfer to the rescue



When a page has no link OUT, treat it as if the page links everywhere. Replace **P** with

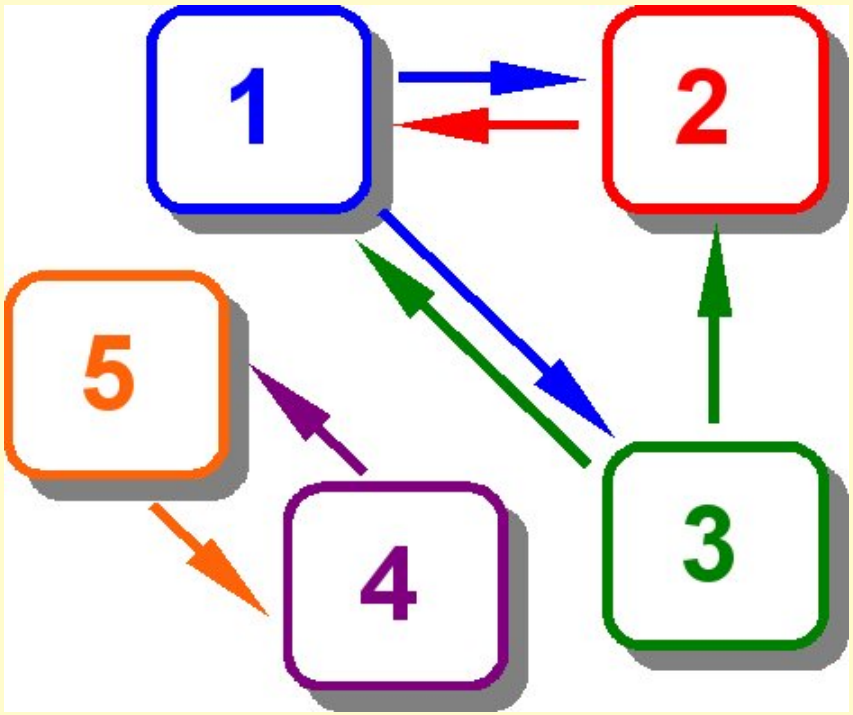
$$\begin{bmatrix} 0 & 1 & 1/2 & 1/4 \\ 1/3 & 0 & 1/2 & 1/4 \\ 1/3 & 0 & 0 & 1/4 \\ 1/3 & 0 & 0 & 1/4 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} \mathbf{R}(p_1) \\ \mathbf{R}(p_2) \\ \mathbf{R}(p_3) \\ \mathbf{R}(p_4) \end{bmatrix} = c \begin{bmatrix} 9/4 \\ 3/2 \\ 1 \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 0.391 \\ 0.261 \\ 0.174 \\ 0.174 \end{bmatrix}$$

Example 3: Disconnected Web

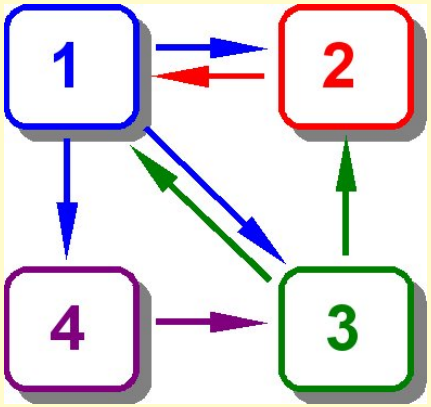
$$\begin{bmatrix} 0 & 1 & 1/2 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{R}(p_1) \\ \mathbf{R}(p_2) \\ \mathbf{R}(p_3) \\ \mathbf{R}(p_4) \\ \mathbf{R}(p_5) \end{bmatrix} = \begin{bmatrix} \mathbf{R}(p_1) \\ \mathbf{R}(p_2) \\ \mathbf{R}(p_3) \\ \mathbf{R}(p_4) \\ \mathbf{R}(p_5) \end{bmatrix}$$

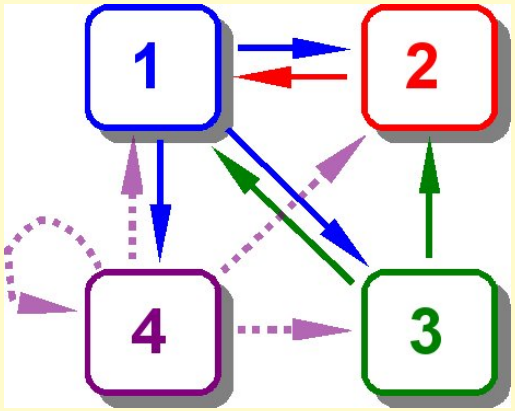


Eigenvectors for $\lambda = 1$:

$$\begin{bmatrix} 0.444 \\ 0.333 \\ 0.222 \\ 0 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.5 \\ 0.5 \end{bmatrix}$$

A little theory





$$\begin{bmatrix} 0 & 1 & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & 0 & 1 \\ \frac{1}{3} & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{3} & 0 & 0 & \frac{1}{4} \\ \frac{1}{3} & 0 & 0 & \frac{1}{4} \end{bmatrix}$$

These had unique solutions

- No dangling nodes!
- Matrices were column stochastic

The Google Matrix

Theorem

For a column stochastic matrix $\mathbf{G}$ *with all positive entries, there exists a unique vector* q *with all positive entries which is also column stochastic so that* $\mathbf{G}q = q$.

Consider an $n \times n$ matrix

$$\mathbf{G} = (1-\alpha)\mathbf{P} + \alpha\mathbf{N}$$

where $N_{ij} = \frac{1}{n}$ and $\mathbf{P}$ has no dangling nodes.

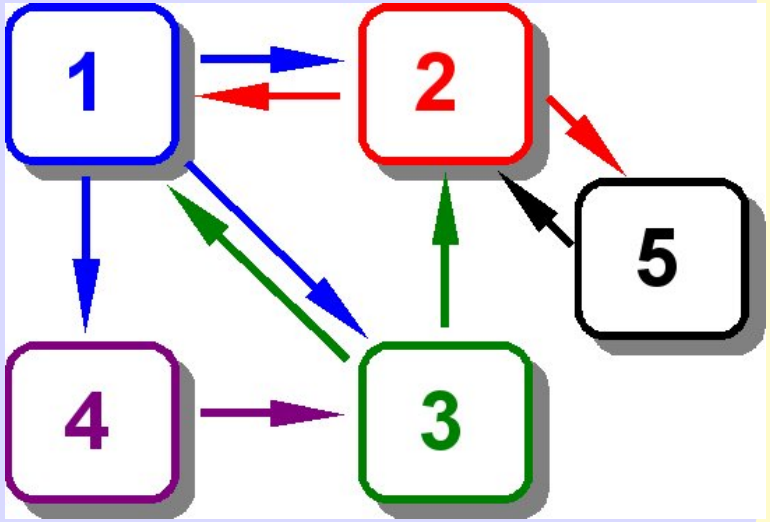
$\mathbf{G}$ guarantees we get a page ranking we can use.

* FACTOID: In 2007 Google altered their search engine ranking system to counteract link bombs.

Mutual Linking

You trade links in attempt to improve your rank. . .

You were tied for 2nd place.
What happens to your ranking?



You will move up! But this doesn't always happen. [Try it.]

Parting Words

Google's efficient and effective algorithm has changed our generation's ability to find info (and entertainment!) we seek.

Still, Challenges Ahead –

- The web is still expanding and evolving at an enormous pace.
- The *type of content* is rapidly shifting away from text based content to flash, video, and other content.
- *Recommendation engines* are another wave of innovation based on powerful linear algebra and numerical analysis techniques.

References


Kurt Bryan & Tanya Leise
The $25,000,000,000 Eigenvector: The Lin.Alg.behind Google
SIAM Review, 48(3):596-581, 2006.


David Austin
How Google Finds Your Needle in the Web's Haystack.
Feature Column at AMS website, Dec 2006.


Michael W. Berry & Murray Browne
Understanding Search Engines: Mathematical Modeling... 2nd Ed.
SIAM, 2005.


John Battelle
The Search: How Google and Its Rivals Rewrote the Rules...
Portfolio, 2005.


Anton and Rorres
Elementary Linear Algebra, Applications Version, 8th Ed.
Wiley & Sons, 2000, pp. 581-583.

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Thank you for listening!

`http://www.LeighNoble.org/`